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Generative Diffusion Prior for Unified Image Restoration and Enhancement

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1. Linear Inverse Image Restoration   Denoising Diffusion Restoration Models

   image deblurring, super-resolution, and compressive sensing, inpainting, colorization

2. Non-linear Image Restoration.

   image low-light enhancement, HDR image recovery,
   JPEG Artifact Correction

3. Blind Image Restoration

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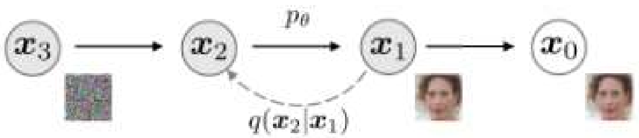

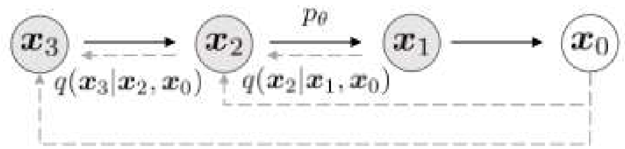

噪声采样 —去噪→ 生成图像

噪声采样 ←加噪— 生成图像

$$\boldsymbol{x}_t = \alpha_t \boldsymbol{x}_{t-1} + \beta_t \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I})$$

$$\boldsymbol{x}_t = \underbrace{(\alpha_t \cdots \alpha_1)}_{\text{记为}\bar{\alpha}_t} \boldsymbol{x}_0 + \underbrace{\sqrt{1 - (\alpha_t \cdots \alpha_1)^2}}_{\text{记为}\bar{\beta}_t} \bar{\boldsymbol{\varepsilon}}_t, \quad \bar{\boldsymbol{\varepsilon}}_t \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I})$$

$$\alpha_t \bar{\beta}_{t-1} \bar{\boldsymbol{\varepsilon}}_{t-1} + \beta_t \boldsymbol{\varepsilon}_t$$

$$\beta_t \bar{\boldsymbol{\varepsilon}}_{t-1} - \alpha_t \bar{\beta}_{t-1} \boldsymbol{\varepsilon}_t$$

$$\mathbb{E}[\boldsymbol{\varepsilon}\boldsymbol{\omega}^\top] = \boldsymbol{0}$$

$$\boldsymbol{\varepsilon}_t = \frac{(\beta_t \boldsymbol{\varepsilon} - \alpha_t \bar{\beta}_{t-1} \boldsymbol{\omega}) \bar{\beta}_t}{\beta_t^2 + \alpha_t^2 \bar{\beta}_{t-1}^2} = \frac{\beta_t \boldsymbol{\varepsilon} - \alpha_t \bar{\beta}_{t-1} \boldsymbol{\omega}}{\bar{\beta}_t}$$

https://kexue.fm/archives/9119

$$\boldsymbol{\mu}(\boldsymbol{x}_t) = \frac{1}{\alpha_t}\left(\boldsymbol{x}_t - \beta_t \boldsymbol{\epsilon}_{\boldsymbol{\theta}}(\boldsymbol{x}_t, t)\right)$$

$$\left\| \boldsymbol{\varepsilon}_t - \boldsymbol{\epsilon}_{\boldsymbol{\theta}}(\bar{\alpha}_t \boldsymbol{x}_0 + \alpha_t \bar{\beta}_{t-1} \bar{\boldsymbol{\varepsilon}}_{t-1} + \beta_t \boldsymbol{\varepsilon}_t, t) \right\|^2$$

$$\begin{aligned} &\mathbb{E}_{\bar{\boldsymbol{\varepsilon}}_{t-1}, \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I})}\left[\left\| \boldsymbol{\varepsilon}_t - \boldsymbol{\epsilon}_{\boldsymbol{\theta}}(\bar{\alpha}_t \boldsymbol{x}_0 + \alpha_t \bar{\beta}_{t-1} \bar{\boldsymbol{\varepsilon}}_{t-1} + \beta_t \boldsymbol{\varepsilon}_t, t) \right\|^2\right] \\ =\,&\mathbb{E}_{\boldsymbol{\omega}, \boldsymbol{\varepsilon} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I})}\left[\left\| \frac{\beta_t \boldsymbol{\varepsilon} - \alpha_t \bar{\beta}_{t-1} \boldsymbol{\omega}}{\bar{\beta}_t} - \boldsymbol{\epsilon}_{\boldsymbol{\theta}}(\bar{\alpha}_t \boldsymbol{x}_0 + \bar{\beta}_t \boldsymbol{\varepsilon}, t) \right\|^2\right] \end{aligned}$$

$$\left\| \boldsymbol{\varepsilon} - \frac{\bar{\beta}_t}{\beta_t} \boldsymbol{\epsilon}_{\boldsymbol{\theta}}(\bar{\alpha}_t \boldsymbol{x}_0 + \bar{\beta}_t \boldsymbol{\varepsilon}, t) \right\|^2$$

$$q\left(\boldsymbol{x}_1, \cdots, \boldsymbol{x}_T \mid \boldsymbol{x}_0\right)=\prod_{t=1}^T q\left(\boldsymbol{x}_t \mid \boldsymbol{x}_{t-1}\right), \tag{1}$$

$$\boldsymbol{x}_t=\sqrt{\bar{\alpha}_t} \boldsymbol{x}_0+\sqrt{1-\bar{\alpha}_t} \boldsymbol{\epsilon}, \tag{2}$$

$$p_{\boldsymbol{\theta}}\left(\boldsymbol{x}_0, \cdots, \boldsymbol{x}_{T-1} \mid \boldsymbol{x}_T\right)=\prod_{t=1}^T p_{\boldsymbol{\theta}}\left(\boldsymbol{x}_{t-1} \mid \boldsymbol{x}_t\right), \quad p_{\boldsymbol{\theta}}\left(\boldsymbol{x}_{t-1} \mid \boldsymbol{x}_t\right)=\mathcal{N}\left(\boldsymbol{x}_{t-1} ; \boldsymbol{\mu}_{\boldsymbol{\theta}}\left(\boldsymbol{x}_t, t\right), \Sigma_\theta \boldsymbol{I}\right) \tag{3}$$

$$\boldsymbol{\mu}_\theta\left(\boldsymbol{x}_t, t\right)=\frac{1}{\sqrt{\alpha_t}}\left(\boldsymbol{x}_t-\frac{\beta_t}{\sqrt{1-\bar{\alpha}_t}} \boldsymbol{\epsilon}_\theta\left(\boldsymbol{x}_t, t\right)\right) \tag{4}$$

$$\tilde{\boldsymbol{x}}_0=\frac{\boldsymbol{x}_t}{\sqrt{\bar{\alpha}_t}}-\frac{\sqrt{1-\bar{\alpha}_t} \boldsymbol{\epsilon}_\theta\left(\boldsymbol{x}_t, t\right)}{\sqrt{\bar{\alpha}_t}} \tag{5}$$

$$q\left(\boldsymbol{x}_{t-1} \mid \boldsymbol{x}_t, \tilde{\boldsymbol{x}}_0\right)=\mathcal{N}\left(\boldsymbol{x}_{t-1} ; \tilde{\boldsymbol{\mu}}_t\left(\boldsymbol{x}_t, \tilde{\boldsymbol{x}}_0\right), \tilde{\beta}_t \mathbf{I}\right),$$
$$\text{where} \quad \tilde{\boldsymbol{\mu}}_t\left(\boldsymbol{x}_t, \tilde{\boldsymbol{x}}_0\right)=\frac{\sqrt{\bar{\alpha}_{t-1}} \beta_t}{1-\bar{\alpha}_t} \tilde{\boldsymbol{x}}_0+\frac{\sqrt{\alpha_t}\left(1-\bar{\alpha}_{t-1}\right)}{1-\bar{\alpha}_t} \boldsymbol{x}_t$$
$$\text{and} \quad \tilde{\beta}_t=\frac{1-\bar{\alpha}_{t-1}}{1-\bar{\alpha}_t} \beta_t \tag{6}$$

$$\boldsymbol{y}=\mathcal{D}(\boldsymbol{x}).$$

$$p_{\boldsymbol{\theta}}\left(\boldsymbol{x}_{t-1} | \boldsymbol{x}_t, \boldsymbol{y}\right).$$

$$\begin{aligned} \log p_{\boldsymbol{\theta}}\left(\boldsymbol{x}_{t-1} | \boldsymbol{x}_t, \boldsymbol{y}\right) & =\log \left(p_{\boldsymbol{\theta}}\left(\boldsymbol{x}_{t-1} | \boldsymbol{x}_t\right) p\left(\boldsymbol{y} | \boldsymbol{x}_t\right)\right)+K_1 \\ & \approx \log p(\boldsymbol{r})+K_2, \end{aligned} \tag{7}$$

$$\boldsymbol{r} \sim \mathcal{N}\left(\boldsymbol{r} ; \boldsymbol{\mu}_{\boldsymbol{\theta}}\left(\boldsymbol{x}_t, t\right)+\Sigma \boldsymbol{g}, \Sigma\right)$$
$$\boldsymbol{g} = \nabla_{\boldsymbol{x}_t} \log p\left(\boldsymbol{y} \mid \boldsymbol{x}_t\right) \quad ; \Sigma=\Sigma_\theta\left(\boldsymbol{x}_t\right)$$

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$$p(\boldsymbol{y} \mid \boldsymbol{x}_t) = \frac{1}{Z} \exp\left(-\left[s\mathcal{L}\left(\mathcal{D}(\boldsymbol{x}_t), \boldsymbol{y}\right) + \lambda\mathcal{Q}(\boldsymbol{x}_t)\right]\right) \quad (8)$$

a heuristic approximation

$\mathcal{L}$: image distance metric

Z : normalization factor

s : a scaling factor controlling the magnitude of guidance

$\mathcal{Q}$: the optional quality enhancement loss

λ : scale factor for adjusting the quality of images

$$\begin{aligned}\log p(\boldsymbol{y} \mid \boldsymbol{x}_t) &= -\log Z - s\mathcal{L}\left(\mathcal{D}(\boldsymbol{x}_t), \boldsymbol{y}\right) - \lambda\mathcal{Q}\left(\boldsymbol{x}_t\right) \\ \nabla_{\boldsymbol{x}_t} \log p(\boldsymbol{y} \mid \boldsymbol{x}_t) &= -s\nabla_{\boldsymbol{x}_t}\mathcal{L}\left(\mathcal{D}(\boldsymbol{x}_t), \boldsymbol{y}\right) - \lambda\nabla_{\boldsymbol{x}_t}\mathcal{Q}\left(\boldsymbol{x}_t\right).\end{aligned} \quad (9)$$

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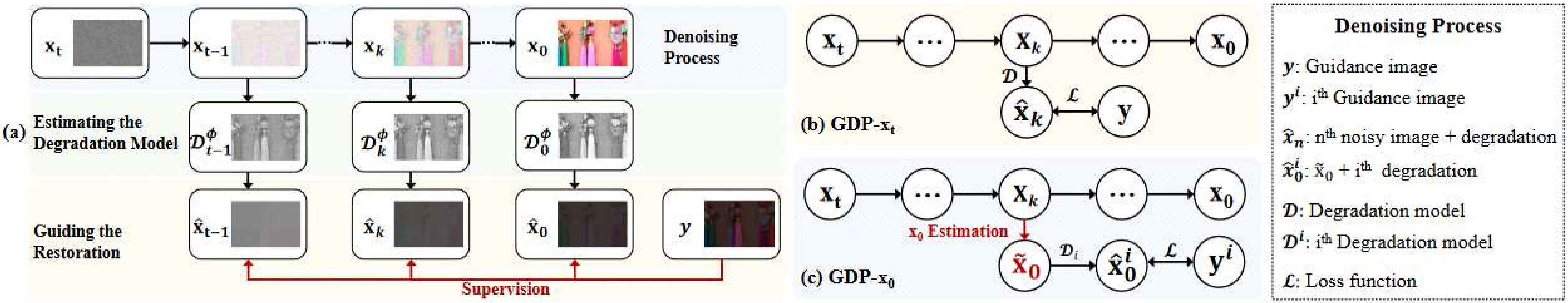

the way of adding guidance and the variance Σ negatively influence the reconstructed images.

The Influence of Variance Σ on the Guidance.

Guidance on xt

Algorithm 1: GDP-x_t with fixed degradation model: Conditioner guided diffusion sampling on x_t, given a diffusion model $(\mu_\theta(x_t), \Sigma_\theta(x_t))$, corrupted image conditioner y.

Input: Corrupted image y, gradient scale s, degradation model $\mathcal{D}$, distance measure $\mathcal{L}$, optional quality enhancement loss $\mathcal{Q}$, quality enhancement scale λ.
Output: Output image x_0 conditioned on y
Sample x_T from $\mathcal{N}(0, \mathbf{I})$
for *t from T to 1* **do**
  $\mu, \Sigma = \mu_\theta(x_t), \Sigma_\theta(x_t)$
  $\mathcal{L}_{x_t}^{total} = \mathcal{L}(y, \mathcal{D}(x_t)) + \mathcal{Q}(x_t)$
  Sample x_{t-1} by $\mathcal{N}(\mu + s\nabla_{x_t}\mathcal{L}_{x_t}^{total}, \Sigma)$
end
return x_0

Guidance on $\tilde{x}_0$.

Algorithm 2: GDP-x_0: Conditioner guided diffusion sampling on $\tilde{x}_0$, given a diffusion model $(\mu_\theta(x_t), \Sigma_\theta(x_t))$, corrupted image conditioner y.

Input: Corrupted image y, gradient scale s, degradation model $\mathcal{D}_\phi$ with randomly initiated parameters ϕ, learning rate l for optimizable degradation model, distance measure $\mathcal{L}$, optional quality enhancement loss $\mathcal{Q}$, quality enhancement scale λ.
Output: Output image x_0 conditioned on y
Sample x_T from $\mathcal{N}(0, \mathbf{I})$
for *t from T to 1* **do**
  $\mu, \Sigma = \mu_\theta(x_t), \Sigma_\theta(x_t)$
  $\tilde{x}_0 = \frac{x_t}{\sqrt{\bar{\alpha}_t}} - \frac{\sqrt{1-\bar{\alpha}_t}\epsilon_\theta(x_t, t)}{\sqrt{\bar{\alpha}_t}}$
  $\mathcal{L}_{\phi,\tilde{x}_0}^{total} = \mathcal{L}(y, \mathcal{D}_\phi(\tilde{x}_0)) + \mathcal{Q}(\tilde{x}_0)$
  $\phi \leftarrow \phi - l\nabla_\phi\mathcal{L}_{\phi,\tilde{x}_0}^{total}$
  Sample x_{t-1} by $\mathcal{N}\left(\mu + s\nabla_{\tilde{x}_0}\mathcal{L}_{\phi,\tilde{x}_0}^{total}, \Sigma\right)$
end
return x_0

$$y = fx + \mathcal{M},$$

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Reconstruction Loss.	Quality Enhancement Loss.
MSE structural similarity index measure perceptual loss	Exposure Control Loss Color Constancy Loss Illumination Smoothness Loss

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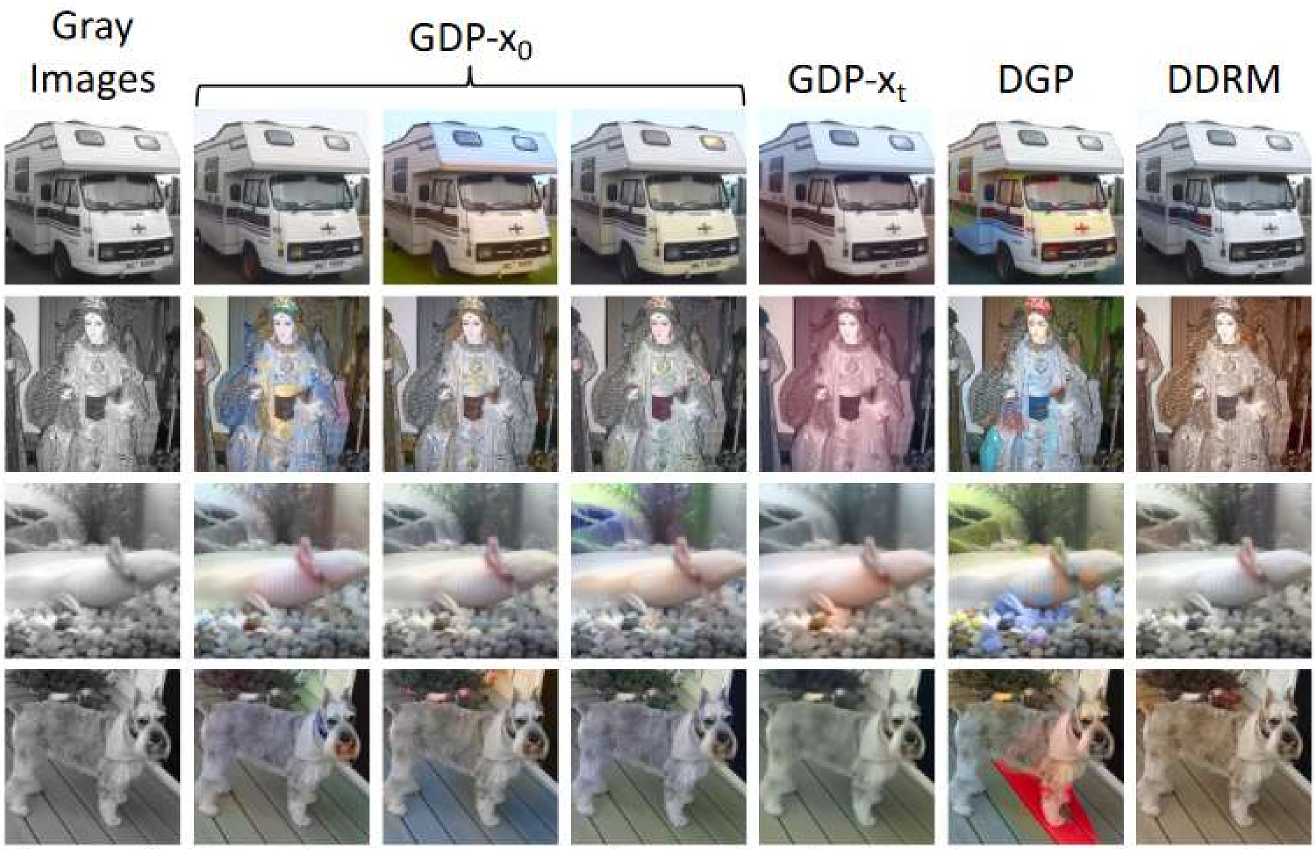

Figure 3. **Qualitative comparison of colorization results on ImageNet validation images**. GDP-x_0 generates various samples on the same input.

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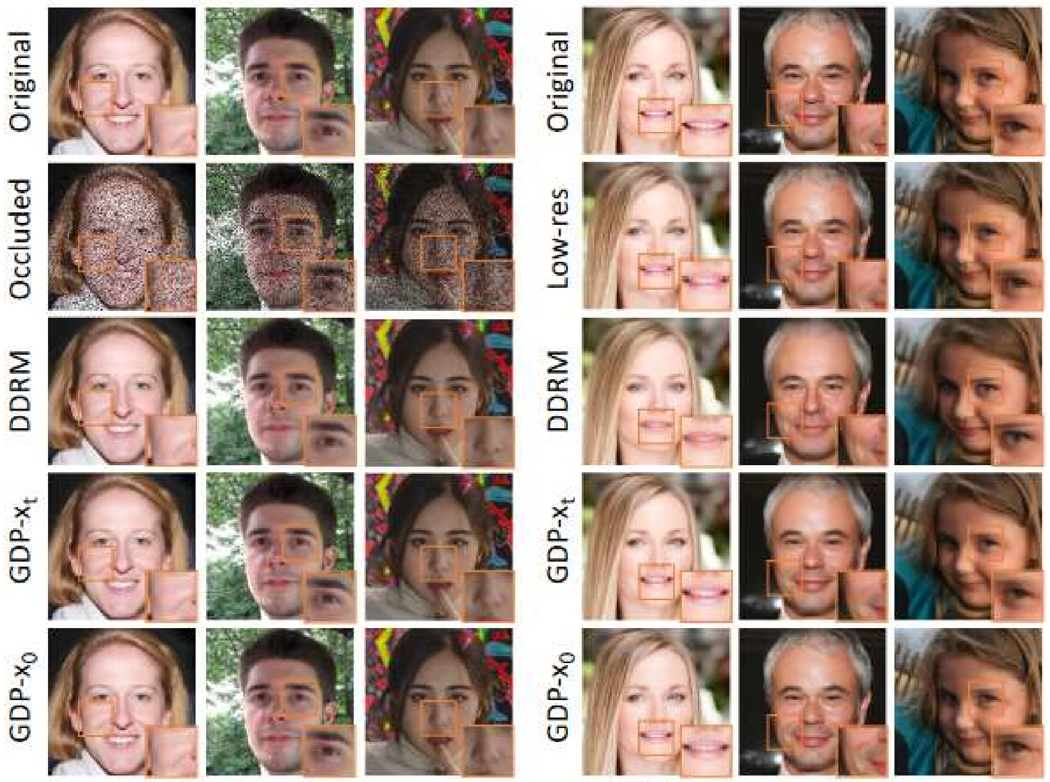
Table 2. **Quantitative comparison of linear image restoration tasks on ImageNet 1k [62].** GDP outperforms other methods in terms of FID and Consistency across all tasks.

Method	4× Super-resolution				Deblur				25% Inpainting				Colorization			
	PSNR ↑	SSIM ↑	Consistency ↓	FID ↓	PSNR ↑	SSIM ↑	Consistency↓	FID ↓	PSNR ↑	SSIM ↑	Consistency↓	FID ↓	PSNR ↑	SSIM ↑	Consistency ↓	FID ↓
DGP [62]	21.65	0.56	158.74	152.85	26.00	0.54	475.10	136.53	27.59	0.82	414.60	60.65	18.42	0.71	305.59	94.59
SNIPS [33]	22.38	0.66	21.38	154.43	24.73	0.69	60.11	17.11	17.55	0.74	587.90	103.50	-	-	-	-
RED [69]	24.18	0.71	27.57	98.30	21.30	0.58	63.20	69.55	-	-	-	-	-	-	-	-
DDRM [32]	**26.53**	**0.78**	19.39	40.75	**35.64**	**0.98**	50.24	4.78	34.28	0.95	**4.08**	24.09	**22.12**	0.91	37.33	47.05
GDP-x_t	24.27	0.67	80.32	64.67	25.86	0.75	54.08	5.00	31.06	0.93	8.80	20.24	21.30	0.86	75.24	66.43
GDP-x_0	24.42	0.68	**6.49**	**38.24**	25.98	0.75	**41.27**	**2.44**	**34.40**	**0.96**	5.29	**16.58**	21.41	**0.92**	**36.92**	**37.60**



(a) Inpainting (b) 4x super resolution

Figure 4. **Qualitative results of (a) 25 % inpainting and (b) 4× super-resolution on CelebA [31].**

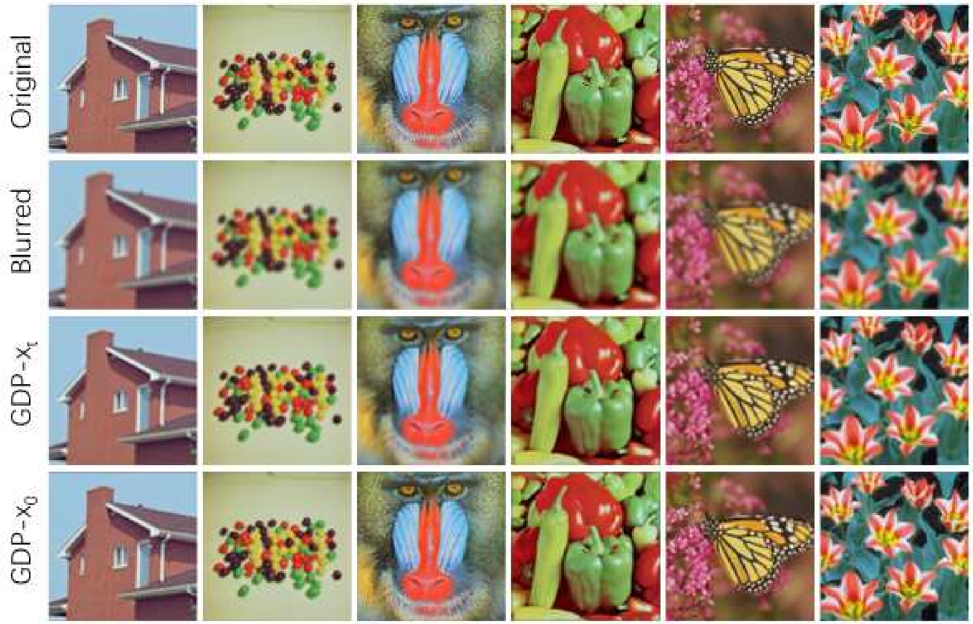


Figure 5. **Results of image deblurring task on 256 × 256 USC-SIPI images [87] using an ImageNet model.**

Figure 6. **Qualitative results of low-light enhancement on the LOL [88], VE-LOL [47], and LoLi-Phone [41] datasets.**

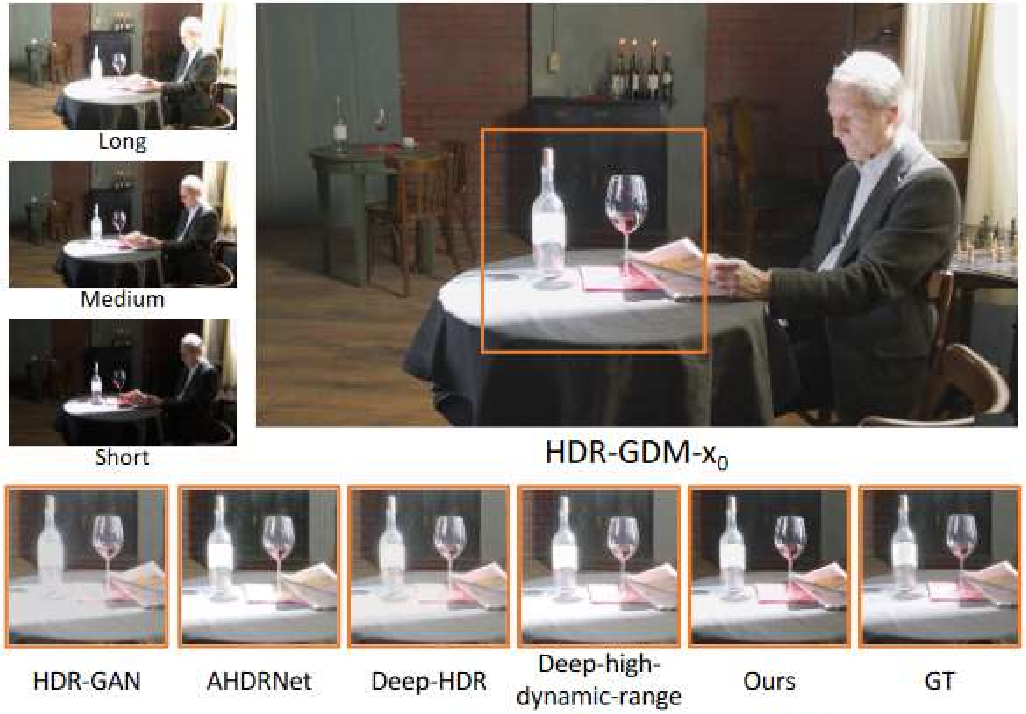


Figure 7. **Example from the NTIRE dataset [63].** We compare a set of patches cropped from the tone-mapped HDR images generated by state-of-the-art methods.

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Table 4. **Quantitative comparison on the NTIRE dataset [63].**

Methods	PSNR↑	SSIM↑	LPIPS ↓	FID↓
AHDRNet [91]	18.72	0.58	0.39	81.98
HDR-GAN [59]	21.67	0.74	0.26	52.71
Deep-HDR [90]	21.66	0.76	0.26	57.52
Deep-high-dynamic-range [30]	21.33	0.71	0.26	51.92
GDP-x_t	19.36	0.65	0.30	63.89
GDP-x_0	**24.88**	**0.86**	**0.13**	**50.05**

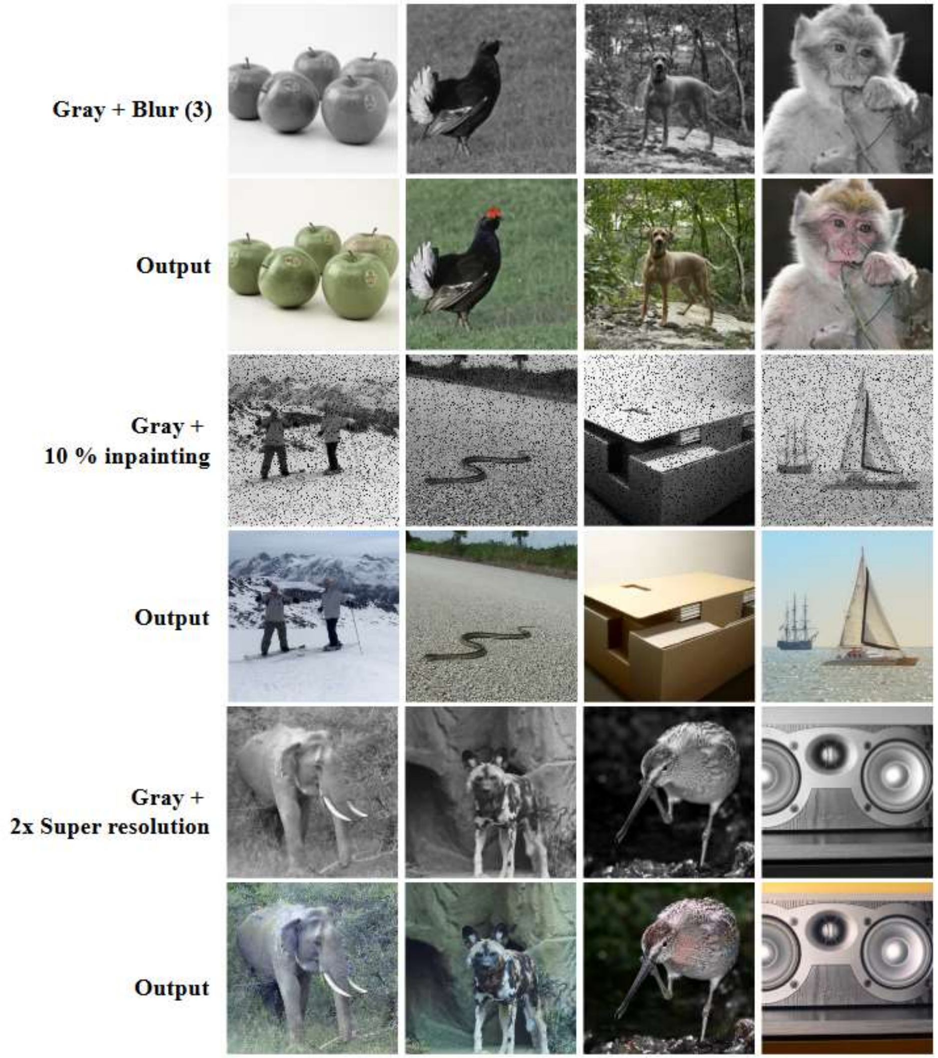
Table 5. **The ablation study on the variance Σ and the way of the guidance.**

Task	4× Super resolution				Deblur			
	PSNR	SSIM	Consistency	FID	PSNR	SSIM	Consistency	FID
GDP -x_t with Σ	22.86	0.60	88.37	68.04	22.06	0.57	69.46	80.39
GDP -x_0 with Σ	22.09	0.58	93.19	41.22	23.49	0.65	68.67	50.29
GDP -x_t	24.27	0.67	80.32	64.67	25.86	0.73	54.08	5.00
GDP -x_0	**24.42**	**0.68**	**6.49**	**38.24**	**25.98**	**0.75**	**41.27**	**2.44**

Task	25% Inpainting				Colorization			
	PSNR	SSIM	Consistency	FID	PSNR	SSIM	Consistency	FID
GDP -x_t with Σ	25.28	0.70	171.44	73.32	17.67	0.70	246.26	145.20
GDP -x_0 with Σ	24.58	0.75	65.59	22.77	21.28	0.91	66.57	38.39
GDP -x_t	31.06	0.93	8.80	20.24	21.30	0.86	75.24	66.43
GDP -x_0	**34.40**	**0.96**	**5.29**	**16.58**	**21.41**	**0.92**	**36.92**	**37.60**

Table 6. **The ablation study on the optimizable degradation and patch-based tactic.**

Methods	LOL					NTIRE			
	PSNR	SSIM	FID	LOE	PI	PSNR	SSIM	LPIPS	FID
Model A	11.05	0.49	156.51	707.57	8.61	24.12	0.67	0.32	86.69
Model B	9.01	0.37	355.99	969.89	9.04	9.83	0.04	1.02	253.11
GDP-x_t	7.32	0.57	238.92	364.15	8.26	19.36	0.65	0.30	63.89
GDP-x_0	**13.93**	**0.63**	**75.16**	**110.39**	**6.47**	**24.88**	**0.86**	**0.13**	**50.05**


Gray + Blur (3)
Output
Gray +
10 % inpainting
Output
Gray +
2x Super resolution
Output

结　　束