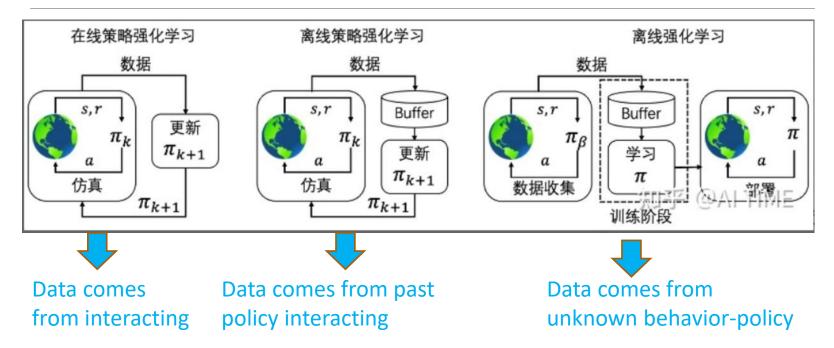
Believe What You See: Implicit Constraint Approach for Offline Multi-Agent Reinforcement Learning

#### NIPS 2021

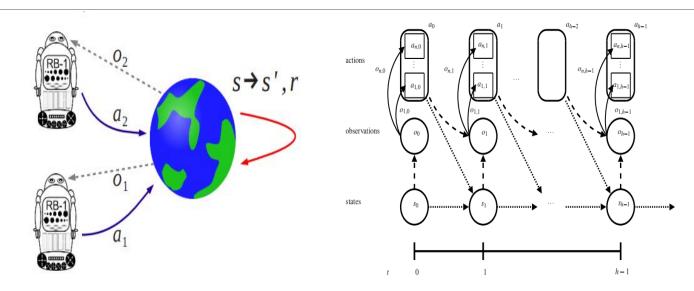
### Offline RL



Offline Advantages:

- 1. Avoid cost of interacting with environment
- 2. Make use of precious expert demonstration

### MULTI-AGENT:dec-POMDP



- 1. This state emits a joint observation
- 2. each agent observes its individual component
- 3. each agent selects an action, together forming the joint action,
- 4. joint action leads to state transition according to the transition model

### Model of Dec-POMDP

#### $G = < S, A, P, r, \Omega, O, n, \gamma >$

S: set of states

A: set of joint actions

P: state transition function

r: reward function shared by all agents

 $\Omega$ : set of joint observations

*O: observation function* 

n: agents

γ: discount rate

# Extrapolation Error

#### **Definition:**

• Extrapolation error is an error in off-policy value learning which is introduced by the mismatch between the dataset and true state-action visitation of the current policy.

#### Cause:

• The extrapolation error mainly attributes the out-of-distribution (OOD) actions in the evaluation of  $Q\pi$  (overestimate the Q value of unknown action)

To quantify the effect of OOD actions, we define the state-action pairs within the dataset as seen pairs. Otherwise, we name them as unseen pairs.

#### Extrapolation Error

$$\epsilon_{\mathrm{EXP}}(\tau, a) = \sum_{\tau'} \left( P_M(\tau' \mid \tau, a) - P_{\mathcal{B}}(\tau' \mid \tau, a) \right) \left( r(\tau, a, \tau') + \gamma \sum_{a'} \pi(a' \mid \tau') Q_{\mathcal{B}}^{\pi}(\tau', a') \right)$$

M: true MDP

B: new MDP computed from batch by P<sub>B</sub> ( $\tau' | \tau$ , a) =  $\mathcal{N}(\tau, a, \tau') / \sum_{\tilde{\tau}} \mathcal{N}(\tau, a, \tilde{\tau})$ .

 $(N(\tau, a, \tau')$  is the number of times the tuple (s, a, s0) is observed inB)

#### E-error in ICQ

Define:

$$\boldsymbol{\epsilon}_{\mathbf{MDP}} = [\boldsymbol{\epsilon}_{\mathbf{s}}, \boldsymbol{\epsilon}_{\mathbf{u}}]^{\mathbf{T}}$$

$$\boldsymbol{\epsilon}_{\mathbf{EXT}} = [\mathbf{0}, \boldsymbol{\epsilon}_{\mathbf{b}}]^{\mathbf{T}}$$

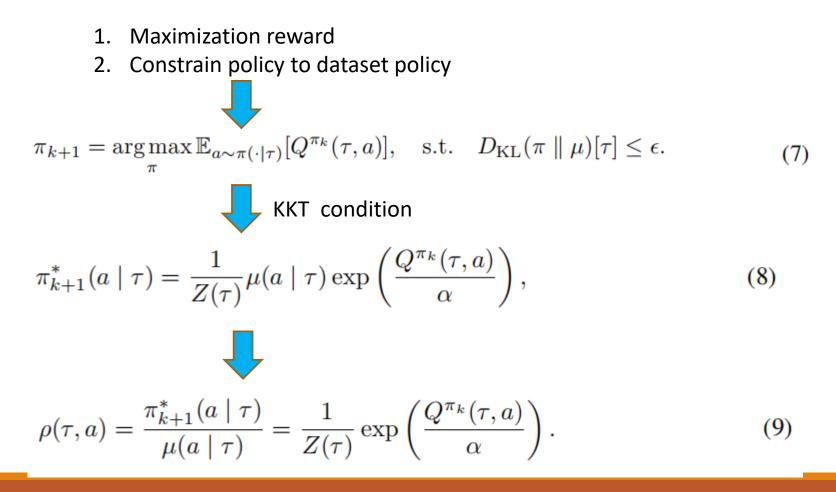
$$P_{M}^{\pi} = \begin{bmatrix} P_{\mathbf{s},\mathbf{s}}^{\pi}, P_{\mathbf{s},\mathbf{u}}^{\pi}; P_{\mathbf{u},\mathbf{s}}^{\pi}, P_{\mathbf{u},\mathbf{u}}^{\pi} \end{bmatrix} \quad \mathbf{c} \qquad \mathbf{c}$$

$$\begin{bmatrix} \boldsymbol{\epsilon}_{\mathbf{s}} \\ \boldsymbol{\epsilon}_{\mathbf{u}} \end{bmatrix} = \gamma \begin{bmatrix} P_{\mathbf{s},\mathbf{s}}^{\pi}, P_{\mathbf{s},\mathbf{u}}^{\pi} \\ P_{\mathbf{u},\mathbf{s}}^{\pi}, P_{\mathbf{u},\mathbf{u}}^{\pi} \end{bmatrix} \begin{bmatrix} \boldsymbol{\epsilon}_{\mathbf{s}} \\ \boldsymbol{\epsilon}_{\mathbf{u}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\epsilon}_{\mathbf{b}} \end{bmatrix}. \quad (3)$$
Es: seen pair estimate error  
Eu: unseen pair estimate error

**Theorem 1.** Given a deterministic MDP, the propagation of  $\epsilon_{\mathbf{b}}$  to  $\epsilon_{\mathbf{s}}$  is proportional to  $\|P_{\mathbf{s},\mathbf{u}}^{\pi}\|_{\infty}$ :

$$\|\boldsymbol{\epsilon}_{\mathbf{s}}\|_{\infty} \leq \frac{\gamma \left\|P_{\mathbf{s},\mathbf{u}}^{\pi}\right\|_{\infty}}{(1-\gamma)\left(1-\gamma \left\|P_{\mathbf{s},\mathbf{s}}^{\pi}\right\|_{\infty}\right)} \|\boldsymbol{\epsilon}_{\mathbf{b}}\|_{\infty}.$$
(4)

#### Implicit Constraint Q-learning



## Implicit Constraint Q-learning

Standard policy evaluation(off policy)  $(\mathcal{T}^{\pi}Q)(\tau,a) \triangleq Q(\tau,a) + \mathbb{E}_{\tau'}[r + \gamma \mathbb{E}_{a' \sim \pi}[Q(\tau',a')] - Q(\tau,a)]. \quad (5)$   $(\mathcal{T}^{\pi}Q)(\tau,a) = Q(\tau,a) + \mathbb{E}_{\tau'}[r + \gamma \mathbb{E}_{a' \sim \mu}[\rho(\tau',a')Q(\tau',a')] - Q(\tau,a)], \quad (6)$ According to Equation 9, it gives the ICQ(re-weight the target value function)

$$\mathcal{T}_{\text{ICQ}}Q(\tau,a) = r + \gamma \mathbb{E}_{a' \sim \mu} \left[ \frac{1}{Z(\tau')} \exp\left(\frac{Q(\tau',a')}{\alpha}\right) Q(\tau',a') \right].$$
(10)

 $Z(\tau) = \sum_{\tilde{a}} \mu(\tilde{a} \mid \tau) \exp\left(\frac{1}{\alpha}Q^{\pi_k}(\tau, \tilde{a})\right)$  is the normalizing partition function

Thus we obtain a SARSA-like algorithm which not uses any unseen pairs.

#### Convergence or No?

**Theorem 2.** Let  $\mathcal{T}_{ICQ}^k Q_0$  denote that the operator  $\mathcal{T}_{ICQ}$  are iteratively applied over an initial stateaction value function  $Q_0$  for k times. Then, we have  $\forall(\tau, a)$ ,  $\limsup_{k\to\infty} \mathcal{T}_{ICQ}^k Q_0(\tau, a) \leq Q^*(\tau, a)$ ,

$$\liminf_{k \to \infty} \mathcal{T}_{\text{ICQ}}^k Q_0(\tau, a) \ge Q^*(\tau, a) - \frac{\gamma(|A| - 1)}{(1 - \gamma)} \max\left\{\frac{1}{(\frac{1}{\alpha} + 1)C + 1}, \frac{2Q_{\max}}{1 + C\exp(\frac{1}{\alpha})}\right\}, \quad (13)$$

where |A| is the action space,  $|A_{\tau}|$  is the action space for state  $\tau$ ,  $C \triangleq \inf_{\tau \in S} \inf_{2 \leq i \leq |A_{\tau}|} \frac{\mu(a_{[1]}|\tau)}{\mu(a_{[i]}|\tau)}$ and  $\mu(a_{[1]} \mid \tau)$  denotes the probability of choosing the expert action according to behavioral policy  $\mu$ . Moreover, the upper bound of  $\mathcal{T}_{BCQ}^k Q_0 - \mathcal{T}_{ICQ}^k Q_0$  decays exponentially fast in terms of  $\alpha$ .

#### ICQ操作符从理论上可以证明收敛到一簇稳定解

### Implicit Constraint Q-learning

Minimizing:

$$\mathcal{J}_Q(\phi) = \mathbb{E}_{\tau, a, \tau', a' \sim \mathcal{B}} \left[ r + \gamma \frac{1}{Z(\tau')} \exp\left(\frac{Q(\tau', a'; \phi')}{\alpha}\right) Q(\tau', a'; \phi') - Q(\tau, a; \phi) \right]^2, \quad (14)$$

Policy learning(minimizing KL distance):

$$\mathcal{J}_{\pi}(\theta) = \mathbb{E}_{\tau \sim \mathcal{B}} \left[ D_{\mathrm{KL}} \left( \pi_{k+1}^{*} \| \pi_{\theta} \right) [\tau] \right] = \mathbb{E}_{\tau \sim \mathcal{B}} \left[ -\sum_{a} \pi_{k+1}^{*} (a \mid \tau) \log \frac{\pi_{\theta}(a \mid \tau)}{\pi_{k+1}^{*}(a \mid \tau)} \right]$$
$$\stackrel{(a)}{=} \mathbb{E}_{\tau \sim \mathcal{B}} \left[ \sum_{a} \frac{\pi_{k+1}^{*}(a \mid \tau)}{\mu(a \mid \tau)} \mu(a \mid \tau) \left( -\log \pi_{\theta}(a \mid \tau) \right) \right]$$
$$\stackrel{(b)}{=} \mathbb{E}_{\tau,a \sim \mathcal{B}} \left[ -\frac{1}{Z(\tau)} \log(\pi(a \mid \tau; \theta)) \exp\left(\frac{Q(\tau, a)}{\alpha}\right) \right],$$

#### ICQ-MA

Value function decompose:

$$\mathcal{J}_{\boldsymbol{\pi}}(\boldsymbol{\theta}) = \sum_{i} \mathbb{E}_{\tau^{i}, a^{i} \sim \mathcal{B}} \left[ -\frac{1}{Z^{i}(\tau^{i})} \log(\pi^{i}(a^{i} \mid \tau^{i}; \boldsymbol{\theta}_{i})) \exp\left(\frac{w^{i}(\boldsymbol{\tau})Q^{i}(\tau^{i}, a^{i})}{\alpha}\right) \right]$$

Value function estimate:

$$\mathcal{J}_{Q}(\phi,\psi) = \mathbb{E}_{\mathcal{B}}\left[\sum_{t\geq 0} (\gamma\lambda)^{t} \left(r_{t} + \gamma \frac{1}{Z(\boldsymbol{\tau}_{t+1})} \exp\left(\frac{Q(\boldsymbol{\tau}_{t+1}, \boldsymbol{a}_{t+1})}{\alpha}\right) Q(\boldsymbol{\tau}_{t+1}, \boldsymbol{a}_{t+1}) - Q(\boldsymbol{\tau}_{t}, \boldsymbol{a}_{t})\right)\right]$$
(19)

where  $Q(\tau_{t+1}, a_{t+1}) = \sum_{i} w^{i}(\tau_{t+1}; \psi') Q^{i}(\tau_{t+1}^{i}, a_{t+1}^{i}; \phi'_{i}) - b(\tau_{t+1}; \psi').$ 

Value estimate with  $\lambda$  return:  $(\mathcal{T}_{ICQ}^{\lambda}Q)(\boldsymbol{\tau}, \boldsymbol{a}) \triangleq Q(\boldsymbol{\tau}, \boldsymbol{a}) + \mathbb{E}_{\mu} \left[ \sum_{t \ge 0} (\gamma \lambda)^{t} (r_{t} + \gamma \rho(\boldsymbol{\tau}_{t+1}, \boldsymbol{a}_{t+1}) Q(\boldsymbol{\tau}_{t+1}, \boldsymbol{a}_{t+1}) - Q(\boldsymbol{\tau}_{t}, \boldsymbol{a}_{t})) \right],$ (20)

#### ICQ IN SINGLE AGENT

Algorithm 1: Implicit Constraint Q-Learning in Single-Agent Tasks.

**Input:** Offline buffer  $\mathcal{B}$ , target network update rate d.

Initialize critic network  $Q^{\pi}(\cdot;\phi)$  and actor network  $\pi(\cdot;\theta)$  with random parameters. Initialize target networks:  $\phi' = \phi$ ,  $\theta' = \theta$ . for t = 1 to T do Sample trajectories from  $\mathcal{B}$ . Train policy according to  $\mathcal{J}_{\pi}(\theta) = \mathbb{E}_{\tau \sim \mathcal{B}} \left[ -\frac{1}{Z(\tau)} \log(\pi(a \mid \tau; \theta)) \exp\left(\frac{Q^{\pi}(\tau, a)}{\alpha}\right) \right]$ . Train critic according to  $\mathcal{J}_{Q}(\phi) = \mathbb{E}_{\tau \sim \mathcal{B}} \left[ r + \gamma \frac{1}{Z(\tau')} \exp\left(\frac{Q(\tau', a'; \phi')}{\alpha}\right) Q(\tau', a'; \phi') - Q(\tau, a; \phi) \right]^{2}$ . if  $t \mod d = 0$  then | Update target networks:  $\phi' = \phi$ ,  $\theta' = \theta$ . end

## ICQ In multi-agent

Algorithm 2: Implicit Constraint Q-Learning in Multi-Agent Tasks.

**Input:** Offline buffer  $\mathcal{B}$ , target network update rate d.

Initialize critic networks  $Q^i(\cdot; \phi_i)$ , actor networks  $\pi^i(\cdot; \theta_i)$  and Mixer network  $M(\cdot; \psi)$  with random parameters. Initialize target networks:  $\phi' = \phi$ ,  $\theta' = \theta$ ,  $\psi' = \psi$ . for t = 1 to T do Sample trajectories from  $\mathcal{B}$ . Train individual policy according to  $\mathcal{J}_{\pi}(\theta) = \sum_i \mathbb{E}_{\tau^i, a^i} \sim_{\mathcal{B}} \left[ -\frac{1}{Z^i(\tau^i)} \log(\pi^i(a^i \mid \tau^i; \theta_i)) \exp\left(\frac{w^i(\tau)Q^i(\tau^i, a^i)}{\alpha}\right) \right]$ . Train critic according to  $\mathcal{J}_Q(\phi, \psi) =$   $\mathbb{E}_{\mathcal{B}} \left[ \sum_{t \ge 0} (\gamma \lambda)^t \left[ r_t + \gamma \frac{\exp\left(\frac{1}{\alpha}Q(\tau_{t+1}, a_{t+1}; \phi', \psi')\right)}{Z(\tau_{t+1}; \phi', \psi')} Q(\tau_{t+1}, a_{t+1}; \phi', \psi') - Q(\tau_t, a_t; \phi, \psi) \right] \right]^2$ . if  $t \mod d = 0$  then | Update target networks:  $\phi' = \phi$ ,  $\theta' = \theta$ ,  $\psi' = \psi$ . end

#### Experiments

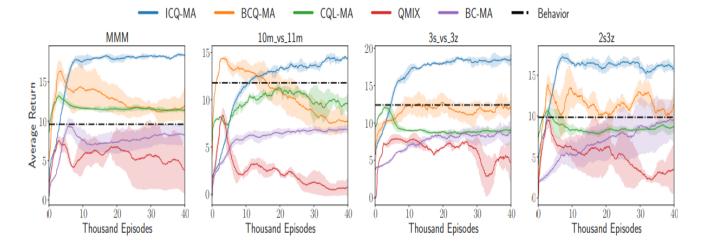


Figure 4: Performance comparison in offline StarCraft II tasks.

## Baselines comparison

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Dataset type	Environment	ICQ (ours)	BC	BCQ	CQL	AWR	BRAC-p
fixed	antmaze-umaze	$85.0\pm2.7$	65.0	78.9	74.0	56.0	50.0
play	antmaze-medium	$80.0 \pm 1.3$	0.0	0.0	61.2	0.0	0.0
play	antmaze-large	$51.0 \pm 4.8$	0.0	6.7	15.8	0.0	0.0
diverse	antmaze-umaze	$65.0 \pm 3.3$	55.0	55.0	84.0	70.3	40.0
diverse	antmaze-medium	$65.0 \pm 3.9$	0.0	0.0	53.7	0.0	0.0
diverse	antmaze-large	$44.0\pm4.2$	0.0	2.2	14.9	0.0	0.0
expert	adroit-door	$103.9\pm3.6$	101.2	99.0	-	102.9	-0.3
expert	adroit-relocate	$109.5 \pm 11.1$	101.3	41.6	-	91.5	-0.3
expert	adroit-pen	$123.8\pm22.1$	85.1	114.9	-	111.0	-3.5
expert	adroit-hammer	$128.3\pm2.5$	125.6	107.2	-	39.0	0.3
human	adroit-door	$6.4{\pm}2.4$	0.5	-0.0	9.1	0.4	-0.3
human	adroit-relocate	$1.5\pm0.7$	-0.0	-0.1	0.35	-0.0	-0.3
human	adroit-pen	$91.3 \pm 10.3$	34.4	68.9	55.8	12.3	8.1
human	adroit-hammer	$2.0{\pm}0.9$	1.5	0.5	<b>2.1</b>	1.2	0.3
medium	walker2d	$71.8 {\pm} 10.7$	66.6	53.1	79.2	17.4	77.5
medium	hopper	$55.6 \pm 5.7$	49.0	54.5	<b>58.0</b>	35.9	32.7
medium	halfcheetah	$42.5 \pm 1.3$	36.1	40.7	<b>44.4</b>	37.4	43.8
med-expert	walker2d	$98.9 \pm 5.2$	66.8	57.5	98.7	53.8	76.9
med-expert	hopper	$109.0 \pm 13.6$	111.9	110.9	111.0	27.1	1.9
med-expert	halfcheetah	$110.3\pm1.1$	35.8	64.7	104.8	52.7	44.2