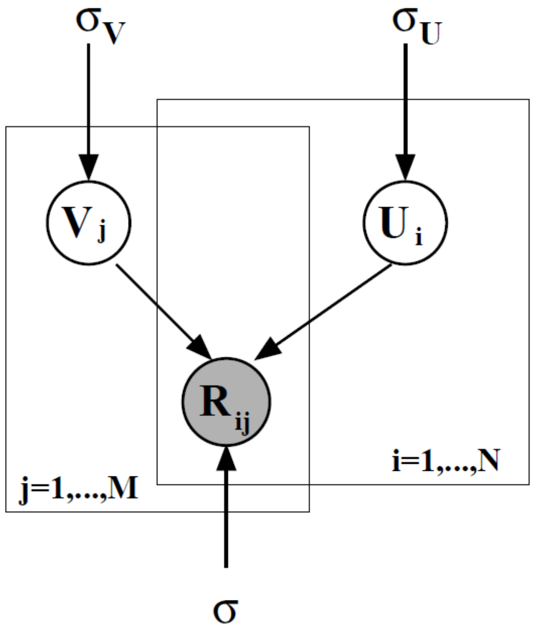
Review

$$\hat{R} = U^T V \qquad U \in R^{D\times N} \qquad V \in R^{D\times M}$$

$$\min_{U,V} \mathcal{L}(R,U,V) = \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n} I_{ij}^R (R_{ij} - g(U_i^T V_j))^2 + \frac{\lambda_U}{2}\|U\|_F^2 + \frac{\lambda_V}{2}\|V\|_F^2$$

$$g(x) = 1/(1+\exp(-x))$$

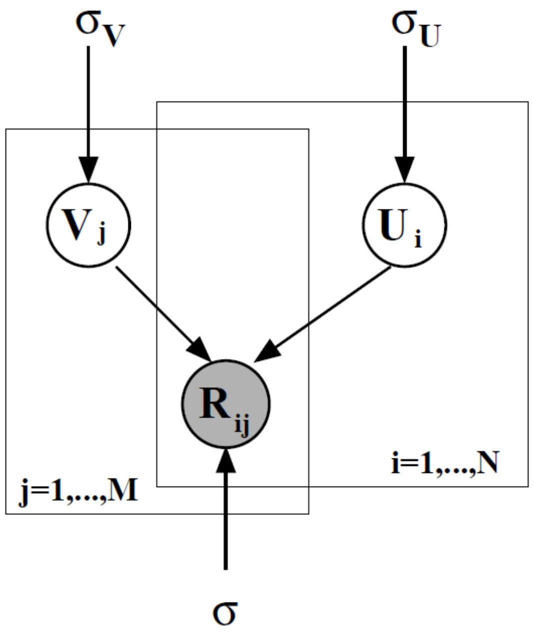
$$p(R|U,V,\sigma^2) = \prod_{i=1}^{N}\prod_{j=1}^{M}\left[\mathcal{N}(R_{ij}|U_i^T V_j, \sigma^2)\right]^{I_{ij}}$$



$$p(V|\sigma_V^2) = \prod_{j=1}^{M}\mathcal{N}(V_j|0,\sigma_V^2\mathbf{I}) \qquad p(U|\sigma_U^2) = \prod_{i=1}^{N}\mathcal{N}(U_i|0,\sigma_U^2\mathbf{I})$$

$$\ln p(U,V|R,\sigma^2,\sigma_V^2,\sigma_U^2) = -\frac{1}{2\sigma^2}\sum_{i=1}^{N}\sum_{j=1}^{M} I_{ij}(R_{ij}-U_i^T V_j)^2 - \frac{1}{2\sigma_U^2}\sum_{i=1}^{N} U_i^T U_i - \frac{1}{2\sigma_V^2}\sum_{j=1}^{M} V_j^T V_j$$
$$-\frac{1}{2}\left(\left(\sum_{i=1}^{N}\sum_{j=1}^{M} I_{ij}\right)\ln\sigma^2 + ND\ln\sigma_U^2 + MD\ln\sigma_V^2\right) + C, \qquad (3)$$

$$\boxed{\begin{array}{c} p(R|U,V,\sigma^2) = \prod_{i=1}^{N}\prod_{j=1}^{M}\left[\mathcal{N}(R_{ij}|U_i^T V_j,\sigma^2)\right]^{I_{ij}} \\ p(V|\sigma_V^2) = \prod_{j=1}^{M}\mathcal{N}(V_j|0,\sigma_V^2\mathbf{I}) \qquad p(U|\sigma_U^2) = \prod_{i=1}^{N}\mathcal{N}(U_i|0,\sigma_U^2\mathbf{I}) \end{array}}$$



$$E = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{M} I_{ij}(R_{ij}-U_i^T V_j)^2 + \frac{\lambda_U}{2}\sum_{i=1}^{N}\| U_i \|_{Fro}^2 + \frac{\lambda_V}{2}\sum_{j=1}^{M}\| V_j \|_{Fro}^2$$

$$\lambda_U = \sigma^2/\sigma_U^2 \qquad \lambda_V = \sigma^2/\sigma_V^2$$

Social Recommendation

↓

Collaborative Topic Regression

2018.10.17

Reference

SoRec: Social Recommendation Using Probabilistic Matrix Factorization CIKM 2008

Learning to Recommend with Social Trust Ensemble SIGIR2009

Recommender Systems with Social Regularization WSDM2011

An Experimental Study on Implicit Social Recommendation SIGIR2013

Exploiting Local and Global Social Context for Recommendation IJCAI2013

Social Collaborative Filtering by Trust IJCAI2013

Collaborative Topic Modeling for Recommending Scientific Articles   KDD2011

Collaborative Topic Regression with Social Matrix Factorization for Recommendation Systems ICML2012

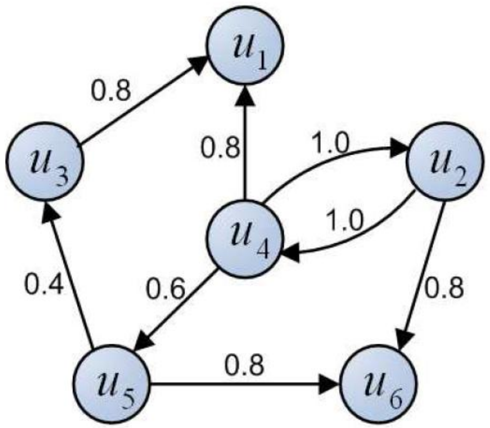
SoRec: Social Recommendation Using Probabilistic Matrix Factorization CIKM 2008

Traditional recommender systems assume that users are i.i.d. (independent and identically distributed); this assumption ignores the social interactions or connections among users.

$$p(C|U,Z,\sigma_C^2) = \prod_{i=1}^{m}\prod_{k=1}^{m}\mathcal{N}\left[\left(c_{ik}|g(U_i^T Z_k),\sigma_C^2\right)\right]^{I_{ik}^C}$$

$$p(U|\sigma_U^2) = \prod_{i=1}^{m}\mathcal{N}(U_i|0,\sigma_U^2\mathbf{I}) \qquad p(Z|\sigma_Z^2) = \prod_{k=1}^{m}\mathcal{N}(Z_k|0,\sigma_Z^2\mathbf{I})$$

$$p(U,Z|C,\sigma_C^2,\sigma_U^2,\sigma_Z^2) \propto p(C|U,Z,\sigma_C^2)p(U|\sigma_U^2)p(Z|\sigma_Z^2)$$



(a) Social Network Graph

$$= \prod_{i=1}^{m}\prod_{k=1}^{m}\mathcal{N}\left[\left(c_{ik}|g(U_i^T Z_k),\sigma_C^2\right)\right]^{I_{ik}^C} \times \prod_{i=1}^{m}\mathcal{N}(U_i|0,\sigma_U^2\mathbf{I}) \times \prod_{k=1}^{m}\mathcal{N}(Z_k|0,\sigma_Z^2\mathbf{I})$$

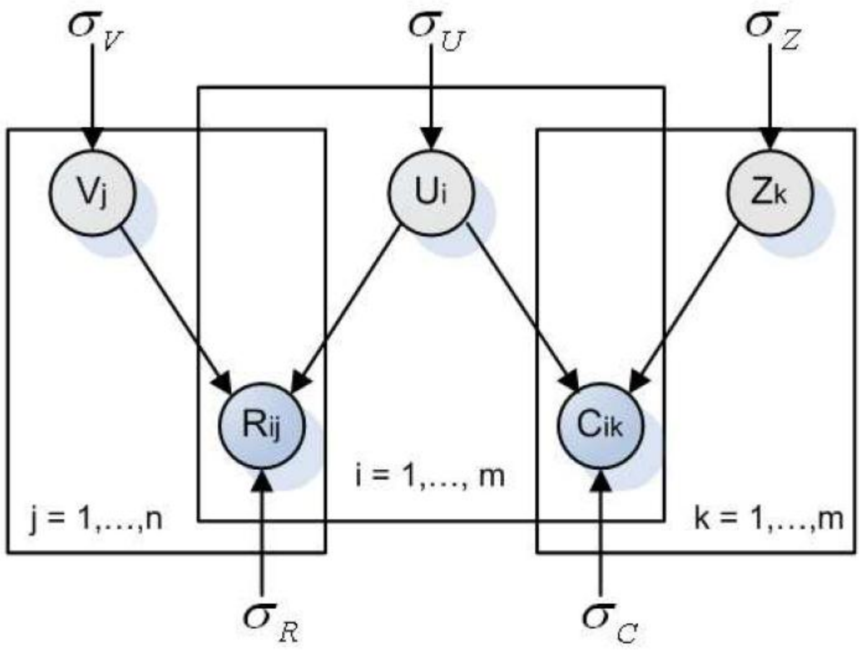
$$p(C|U,Z,\sigma_C^2) = \prod_{i=1}^{m}\prod_{j=1}^{n}\mathcal{N}\left[\left(c_{ik}^*|g(U_i^T Z_k),\sigma_C^2\right)\right]^{I_{ik}^C}$$

$$c_{ik}^* = \sqrt{\frac{d^-(v_k)}{d^+(v_i)+d^-(v_k)}} \times c_{ik}$$

$$p(R|U,V,\sigma_R^2) = \prod_{i=1}^{m}\prod_{j=1}^{n}\mathcal{N}\left[\left(r_{ij}|g(U_i^T V_j),\sigma_R^2\right)\right]^{I_{ij}^R}$$

$$p(U|\sigma_U^2) = \prod_{i=1}^{m}\mathcal{N}(U_i|0,\sigma_U^2\mathbf{I}) \qquad p(V|\sigma_V^2) = \prod_{j=1}^{n}\mathcal{N}(V_j|0,\sigma_V^2\mathbf{I})$$

$$p(U,V|R,\sigma_R^2,\sigma_U^2,\sigma_V^2) \quad \propto \quad p(R|U,V,\sigma_R^2)p(U|\sigma_U^2)p(V|\sigma_V^2)$$



$$p(U,V,Z|C,R,\sigma_C^2,\sigma_R^2,\sigma_U^2,\sigma_V^2,\sigma_Z^2)$$

$$\mathcal{L}(R,C,U,V,Z) = \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n} I_{ij}^R(r_{ij}-g(U_i^T V_j))^2 + \frac{\lambda_C}{2}\sum_{i=1}^{m}\sum_{k=1}^{m} I_{ik}^C(c_{ik}^*-g(U_i^T Z_k))^2$$

$$+\frac{\lambda_U}{2}\|U\|_F^2 + \frac{\lambda_V}{2}\|V\|_F^2 + \frac{\lambda_Z}{2}\|Z\|_F^2$$

Learning to Recommend with Social Trust Ensemble SIGIR2009

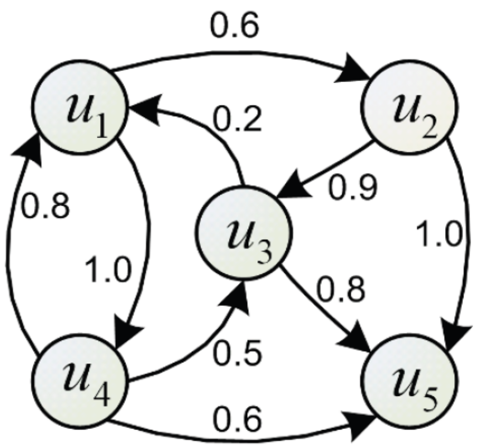
Propose a novel probabilistic factor analysis framework, which naturally fuses the users' tastes and their trusted friends' favors together.

Recommendations by Trusted Friends

$$\widehat{R}_{ik} = \frac{\sum_{j \in \mathcal{T}(i)} R_{jk} S_{ij}}{|\mathcal{T}(i)|} \qquad \widehat{R}_{ik} = \sum_{j \in \mathcal{T}(i)} R_{jk} S_{ij}$$

$$\begin{pmatrix} \widehat{R}_{i1} \\ \widehat{R}_{i2} \\ \dots \\ \widehat{R}_{in} \end{pmatrix} = \begin{pmatrix} R_{11} & R_{21} & \dots & R_{m1} \\ R_{12} & R_{22} & \dots & R_{m2} \\ \dots & \dots & \dots & \dots \\ R_{1n} & R_{2n} & \dots & R_{mn} \end{pmatrix} \begin{pmatrix} S_{i1} \\ S_{i2} \\ \dots \\ S_{im} \end{pmatrix}$$

$$\widehat{R} = SR$$



(a) Social Trust Graph

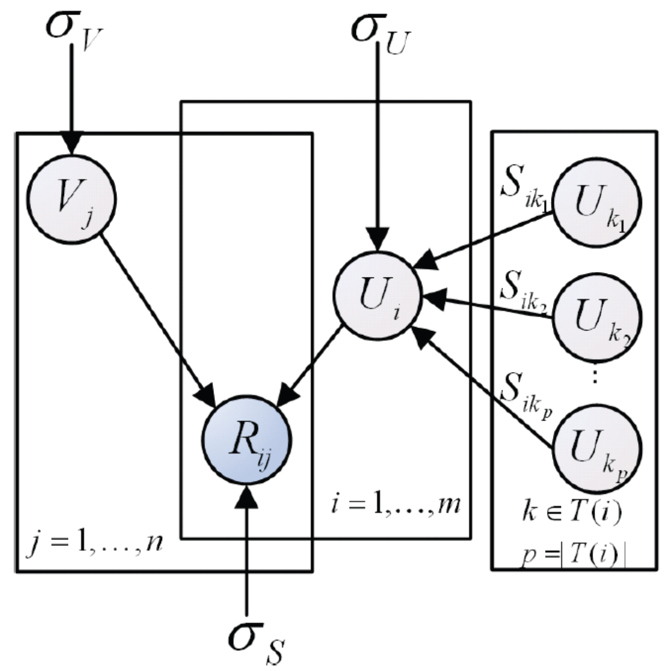
	v_1	v_2	v_3	v_4	v_5	v_6
u_1		5	2		3	
u_2	4			3		4
u_3			2			2
u_4	5			3		
u_5		5	5			3

(b) User-Item Rating Matrix

$$p(R|S,U,V,\sigma_R^2) = \prod_{i=1}^{m}\prod_{j=1}^{n}\left[\mathcal{N}\left(R_{ij}|g(\sum_{k\in\mathcal{T}(i)} S_{ik}U_k^TV_j),\sigma_S^2\right)\right]^{I_{ij}^R}$$

$$p(U,V|R,S,\sigma_S^2,\sigma_U^2,\sigma_V^2) \propto p(R|S,U,V,\sigma_S^2)p(U|S,\sigma_U^2)p(V|S,\sigma_V^2)$$

We can assume that S is independent with the low-dimensional matrices U and V.

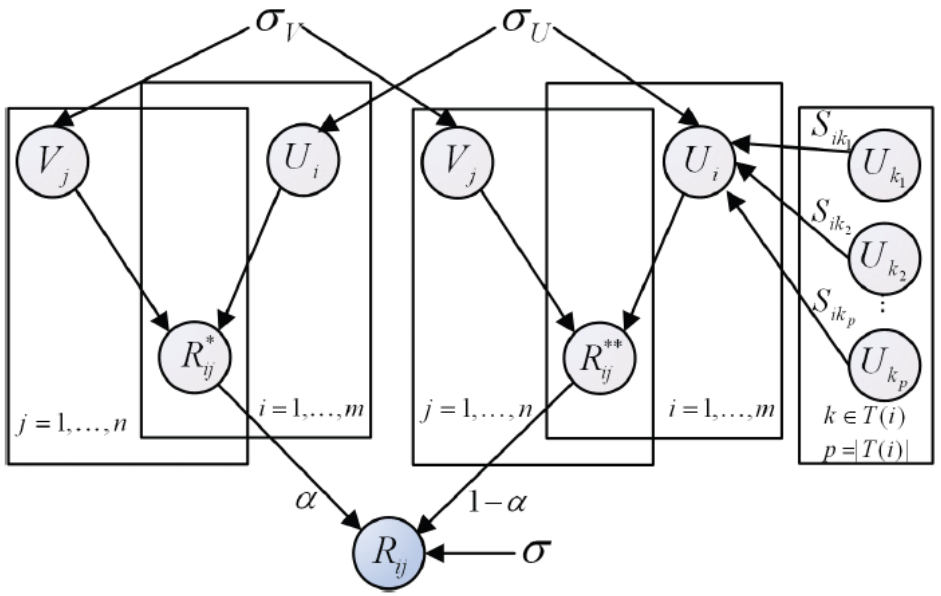


(b) Recommendations by Trusted Friends

$$p(U,V|R,S,\sigma_S^2,\sigma_U^2,\sigma_V^2) \propto p(R|S,U,V,\sigma_S^2)p(U|\sigma_U^2)p(V|\sigma_V^2)$$

$$= \prod_{i=1}^{m}\prod_{j=1}^{n}\left[\mathcal{N}\left(R_{ij}|g(\sum_{k\in\mathcal{T}(i)} S_{ik}U_k^TV_j),\sigma_S^2\right)\right]^{I_{ij}^R} \times \prod_{i=1}^{m}\mathcal{N}(U_i|0,\sigma_U^2\mathbf{I}) \times \prod_{j=1}^{n}\mathcal{N}(V_j|0,\sigma_V^2\mathbf{I})$$

$$p(R|U,V,\sigma_R^2) = \prod_{i=1}^{m}\prod_{j=1}^{n}\left[\mathcal{N}\left(R_{ij}|g(U_i^T V_j),\sigma_R^2\right)\right]^{I_{ij}^R}$$



(c) Recommendations with Social Trust Ensemble

$$p(U,V|R,S,\sigma^2,\sigma_U^2,\sigma_V^2)$$

$$= \prod_{i=1}^{m}\prod_{j=1}^{n}\left[\mathcal{N}\left(R_{ij}|g(\alpha U_i^T V_j + (1-\alpha)\sum_{k\in\mathcal{T}(i)} S_{ik}U_k^T V_j),\sigma^2\right)\right]^{I_{ij}^R} \times \prod_{i=1}^{m}\mathcal{N}(U_i|0,\sigma_U^2\mathbf{I}) \times \prod_{j=1}^{n}\mathcal{N}(V_j|0,\sigma_V^2\mathbf{I})$$

$$\mathcal{L}(R,S,U,V) = \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n} I_{ij}^R(R_{ij} - g(\alpha U_i^T V_j + (1-\alpha)\sum_{k\in\mathcal{T}(i)} S_{ik}U_k^T V_j))^2 + \frac{\lambda_U}{2}\|U\|_F^2 + \frac{\lambda_V}{2}\|V\|_F^2$$

Average-based Regularization

$$\min_{U,V} \mathcal{L}_1(R,U,V) = \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n} I_{ij}(R_{ij} - U_i^T V_j)^2 + \frac{\alpha}{2}\sum_{i=1}^{m} \|U_i - \frac{1}{|\mathcal{F}^+(i)|}\sum_{f\in\mathcal{F}^+(i)} U_f\|_F^2 + \frac{\lambda_1}{2}\|U\|_F^2 + \frac{\lambda_2}{2}\|V\|_F^2$$

$$\frac{\alpha}{2}\sum_{i=1}^{m} \|U_i - \frac{1}{|\mathcal{F}^+(i)|}\sum_{f\in\mathcal{F}^+(i)} U_f\|_F^2$$

$$\frac{\alpha}{2}\sum_{i=1}^{m} \|U_i - \frac{\sum_{f\in\mathcal{F}^+(i)} Sim(i,f)\times U_f}{\sum_{f\in\mathcal{F}^+(i)} Sim(i,f)}\|_F^2$$

Individual-based Regularization

$$\frac{\beta}{2}\sum_{i=1}^{m}\sum_{f\in\mathcal{F}^+(i)} Sim(i,f)\|U_i - U_f\|_F^2$$

$$\begin{aligned}\min_{U,V}\mathcal{L}_2(R,U,V) \quad &= \quad \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n} I_{ij}(R_{ij}-U_i^TV_j)^2 + \quad \frac{\beta}{2}\sum_{i=1}^{m}\sum_{f\in\mathcal{F}^+(i)} Sim(i,f)\|U_i - U_f\|_F^2 \\ &+ \quad \lambda_1\|U\|_F^2 + \lambda_2\|V\|_F^2\end{aligned}$$

It indirectly models the propagation of tastes.

$$Sim(i,f)||U_i - U_f||_F^2 \text{ and } Sim(f,g)||U_f - U_g||_F^2$$

$$Sim(i,f) = \frac{\sum_{j\in I(i)\cap I(f)} R_{ij}\cdot R_{fj}}{\sqrt{\sum_{j\in I(i)\cap I(f)} R_{ij}^2}\cdot\sqrt{\sum_{j\in I(i)\cap I(f)} R_{fj}^2}}$$

$$Sim(i,f) = \frac{\sum_{j\in I(i)\cap I(f)} (R_{ij}-\overline{R}_i)\cdot(R_{fj}-\overline{R}_f)}{\sqrt{\sum_{j\in I(i)\cap I(f)} (R_{ij}-\overline{R}_i)^2}\cdot\sqrt{\sum_{j\in I(i)\cap I(f)} (R_{fj}-\overline{R}_f)^2}}$$

An Experimental Study on Implicit Social Recommendation SIGIR2013

Explicit social information is not always available in most of the recommender systems, which limits the impact of social recommendation techniques.

Implicit User Social Relationships

In the case of missing explicit social information, **we can always compute another set of Top-N similar users and then plug in those similar users** to the aforementioned social recommendation matrix factorization framework.

$$L = \min_{U,V} \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} I_{ij} (r_{ij} - \mathbf{u}_i^T \mathbf{v}_j)^2 + \frac{\alpha}{2} \sum_{i=1}^{m} \sum_{f \in \mathcal{F}^+(i)} s_{if} \|\mathbf{u}_i - \mathbf{u}_f\|_F^2 + \frac{\lambda_1}{2} \|U\|_F^2 + \frac{\lambda_2}{2} \|V\|_F^2$$

$$s_{if} = \frac{\sum_{k \in I(i) \cap I(f)} (r_{ik} - \overline{r}_i) \cdot (r_{fk} - \overline{r}_f)}{\sqrt{\sum_{k \in I(i) \cap I(f)} (r_{ik} - \overline{r}_i)^2} \cdot \sqrt{\sum_{k \in I(i) \cap I(f)} (r_{fk} - \overline{r}_f)^2}}$$

Item Social Relationships

$$L = \min_{U,V} \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n} I_{ij}(r_{ij} - \mathbf{u}_i^T \mathbf{v}_j)^2 + \frac{\beta}{2}\sum_{j=1}^{n}\sum_{q\in\mathcal{Q}^+(j)} s_{jq}\|\mathbf{v}_j - \mathbf{v}_q\|_F^2 + \frac{\lambda_1}{2}\|U\|_F^2 + \frac{\lambda_2}{2}\|V\|_F^2$$

$$s_{jq} = \frac{\sum_{k\in U(j)\cap U(q)} (r_{kj} - \overline{r}_j)\cdot(r_{kq} - \overline{r}_q)}{\sqrt{\sum_{k\in U(j)\cap U(q)} (r_{kj} - \overline{r}_j)^2}\cdot\sqrt{\sum_{k\in U(j)\cap U(q)} (r_{kq} - \overline{r}_q)^2}}$$

A Unified Model

$$L = \min_{U,V} \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n} I_{ij}(r_{ij} - \mathbf{u}_i^T \mathbf{v}_j) + \frac{\alpha}{2}\sum_{i=1}^{m}\sum_{f\in\mathcal{F}^+(i)} s_{if}\|\mathbf{u}_i - \mathbf{u}_f\|_F^2$$

$$+ \frac{\beta}{2}\sum_{j=1}^{n}\sum_{q\in\mathcal{Q}^+(j)} s_{jq}\|\mathbf{v}_j - \mathbf{v}_q\|_F^2 + \frac{\lambda_1}{2}\|U\|_F^2 + \frac{\lambda_2}{2}\|V\|_F^2$$

Exploiting Local Social Context

Since users with strong ties are more likely to share similar tastes than those with weak ties, treating all social relations equally is likely to lead to degradation in recommendation performance.

$$\min \sum_{\langle u_i, v_j \rangle \in \mathcal{O}} (\mathbf{R}_{ij} - \mathbf{u}_i^\top \mathbf{v}_j)^2 + \lambda(\|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2)$$

$$\mathbf{S}_{ik} = \begin{cases} \frac{\sum_j \mathbf{R}_{ij} \cdot \mathbf{R}_{kj}}{\sqrt{\sum_j \mathbf{R}_{ij}^2}\sqrt{\sum_j \mathbf{R}_{kj}^2}} & \text{for } u_k \in \mathcal{N}_i, \\ 0 & \text{for } u_k \notin \mathcal{N}_i. \end{cases}$$

$$\min \sum_{i=1}^{n} \sum_{u_k \in \mathcal{N}_i} (\mathbf{S}_{ik} - \mathbf{u}_i^\top \mathbf{H} \mathbf{u}_k)^2$$

$$\min \|\mathbf{T} \odot (\mathbf{S} - \mathbf{U}^\top \mathbf{H} \mathbf{U})\|_F^2$$

Exploiting Global Social Context

The solution to capture global social context is to weight the importance of user ratings according to their reputation scores。

PageRank

$$\min \sum_{\langle u_i, v_j \rangle \in \mathcal{O}} w_i (\mathbf{R}_{ij} - \mathbf{u}_i^\top \mathbf{v}_j)^2 \qquad w_i = f(r_i) = \frac{1}{1 + log(r_i)} \qquad r_i \in [1, n]$$

$$\min \|\mathbf{W} \odot (\mathbf{R} - \mathbf{U}^\top \mathbf{V})\|_F^2 \qquad \mathbf{W}_{ij} = \begin{cases} \sqrt{w_i} & \text{if } \mathbf{R}_{ij} \neq 0 \\ 0 & \text{if } \mathbf{R}_{ij} = 0 \end{cases}$$

$$\min_{\mathbf{U},\mathbf{V},\mathbf{H}} \sum_{\langle u_i, v_j \rangle \in \mathcal{O}} w_i (\mathbf{R}_{ij} - \mathbf{u}_i^\top \mathbf{v}_j)^2 + \alpha \sum_{i=1}^{n} \sum_{u_k \in \mathcal{N}_i} (\mathbf{S}_{ik} - \mathbf{u}_i^\top \mathbf{H} \mathbf{u}_k)^2 + \lambda(\|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2 + \|\mathbf{H}\|_F^2)$$

$$\mathcal{J} = \|\mathbf{W} \odot (\mathbf{R} - \mathbf{U}^\top \mathbf{V})\|_F^2 + \alpha \|\mathbf{T} \odot (\mathbf{S} - \mathbf{U}^\top \mathbf{H} \mathbf{U})\|_F^2 + \lambda(\|\mathbf{U}\|_F^2 + \|\mathbf{V}\|_F^2 + \|\mathbf{H}\|_F^2)$$

$$\min \sum_{i=1}^{n} \sum_{u_k \in \mathcal{N}_i} (\mathbf{S}_{ik} - \mathbf{u}_i^\top \mathbf{z}_k)^2 \qquad \min \sum_{i=1}^{n} \sum_{u_k \in \mathcal{N}_i} (\mathbf{S}_{ik} - \mathbf{u}_i^\top \mathbf{H} \mathbf{u}_k)^2$$

$$\min \sum_{i=1}^{n} \sum_{u_k \in \mathcal{N}_i} (\mathbf{S}_{ik} - \mathbf{u}_i^\top \mathbf{z}_k)^2$$

$$\min \sum_{i=1}^{n} \sum_{u_k \in \mathcal{N}_i} (\mathbf{S}_{ik} - \mathbf{u}_i^\top \mathbf{H} \mathbf{u}_k)^2$$

$$\mathcal{L}(R, C, U, V, Z) = \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} I_{ij}^{R} (r_{ij} - g(U_i^T V_j))^2 + \frac{\lambda_C}{2} \sum_{i=1}^{m} \sum_{k=1}^{m} I_{ik}^{C} (c_{ik}^{*} - g(U_i^T Z_k))^2 + \frac{\lambda_U}{2} \|U\|_F^2 + \frac{\lambda_V}{2} \|V\|_F^2 + \frac{\lambda_Z}{2} \|Z\|_F^2$$

$$\min \sum_{i=1}^{n} \sum_{u_k \in \mathcal{N}_i} \mathbf{S}_{ik} \|\mathbf{u}_i - \mathbf{u}_k\|_2^2$$

$$\min \sum_{i=1}^{n} \sum_{u_k \in \mathcal{N}_i} -2\mathbf{S}_{ik} \mathbf{u}_i^\top \mathbf{A} \mathbf{u}_k + \mathbf{u}_i^\top \mathbf{B}_k \mathbf{u}_i + \mathbf{u}_k^\top \mathbf{C}_i \mathbf{u}_k$$

$$\mathbf{A} = \mathbf{I},\ \mathbf{B}_k = \mathbf{C}_i = \frac{1}{\mathbf{S}_{ik}} diag(\mathbf{S}_{ik})$$

$$\mathbf{A} = \mathbf{H},\ \mathbf{B}_k = \frac{1}{2} \mathbf{H} \mathbf{u}_k \mathbf{u}_k^\top \mathbf{H}^\top,\ \mathbf{C}_i = \frac{1}{2} \mathbf{H}^\top \mathbf{u}_i \mathbf{u}_i^\top \mathbf{H}$$

Social Collaborative Filtering by Trust   IJCAI2013

$$\mathcal{L} = \sum\nolimits_{(i,j)\in\Omega}\left(U_i^T V_j - R_{ij}\right)^2 + \lambda\left(\sum\nolimits_i n_{u_i}\|U_i\|_F^2 + \sum\nolimits_j n_{v_j}\left\|V_j\right\|_F^2\right)$$

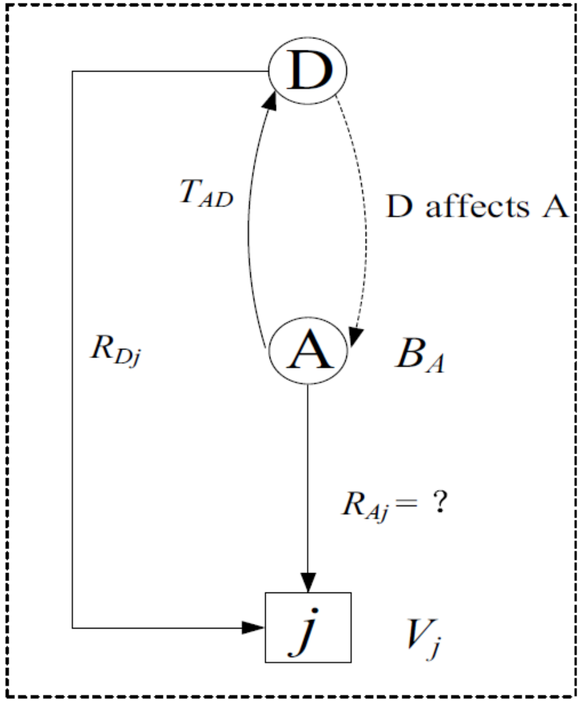
Matrix Factorization of the Trust Network

$$\mathcal{L} = \sum\nolimits_{(i,k)\in\Psi}\left(B_i^T W_k - T_{ik}\right)^2 + \lambda(\|B\|_F^2 + \|W\|_F^2)$$

Truster Model

$$\mathcal{L} = \sum\nolimits_{(i,j)\in\Omega}\left(B_i^T V_j - R_{ij}\right)^2 + \lambda_T\sum\nolimits_{(i,k)\in\Psi}\left(B_i^T W_k - T_{ik}\right)^2 + \lambda(\|B\|_F^2 + \|V\|_F^2 + \|W\|_F^2)$$

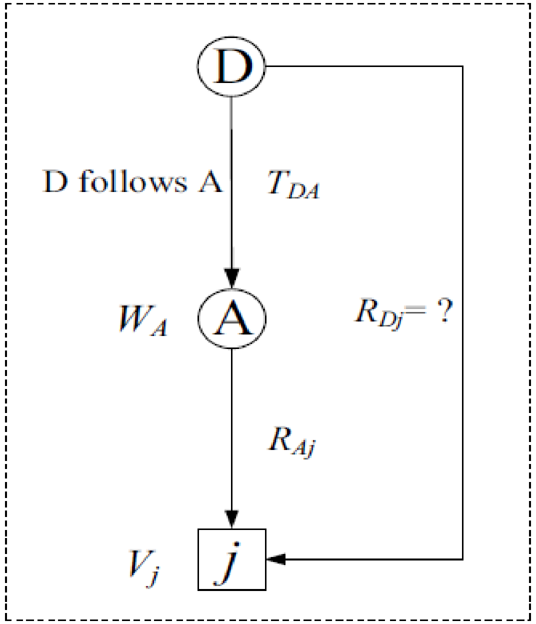
$$\mathcal{L} = \sum_{(i,j)\in\Omega}\left(g(B_i^T V_j) - R_{ij}\right)^2 + \lambda_T\sum_{(i,k)\in\Psi}\left(g(B_i^T W_k) - T_{ik}\right)^2 + \lambda\left(\sum\nolimits_i (n_{b_i} + m_{b_i})\|B_i\|_F^2 + \sum\nolimits_j n_{v_j}\left\|V_j\right\|_F^2 + \sum\nolimits_k m_{w_k}\|W_k\|_F^2\right)$$



Trustee Model

Distinct from truster model, this time we choose the trustee-specific feature matrix ***W*** as the latent space commonly shared by ***R*** and ***T***.

$$\mathcal{L} = \sum_{(i,j)\in\Omega} \left(g(W_i^T V_j) - R_{ij}\right)^2 + \lambda_T \sum_{(k,i)\in\Psi} \left(g(B_k^T W_i) - T_{ki}\right)^2 + \lambda \left(\sum_i (n_{w_i} + m_{w_i}) \|W_i\|_F^2 + \sum_j n_{v_j} \left\|V_j\right\|_F^2 + \sum_k m_{b_k} \|B_k\|_F^2\right)$$



Synthetic Influence of Trust Propagation

$$\hat{R}_{ij} = g\left(\left(\frac{B_i^r + W_i^e}{2}\right)^T \left(\frac{V_j{}^r + V_j{}^e}{2}\right)\right) \cdot R_{max}$$

V_j^r be the item-specific vector learned by algorithm Truster-MF. V_j^e be the item-specific vector learned by algorithm Truster-MF.

the $W_A^T V_j$ indicates how other users follow user A to rate item j, which is the approximation of real score R_{ij} as well.

Collaborative Topic Regression

Collaborative Topic Modeling for Recommending Scientific Articles KDD2011

PMF

$$\min_{U,V} \sum_{i,j}(r_{ij} - u_i^T v_j)^2 + \lambda_u ||u_i||^2 + \lambda_v ||v_j||^2$$

1. For each user i, draw user latent vector $u_i \sim \mathcal{N}(0, \lambda_u^{-1} I_K)$.
2. For each item j, draw item latent vector $v_j \sim \mathcal{N}(0, \lambda_v^{-1} I_K)$.

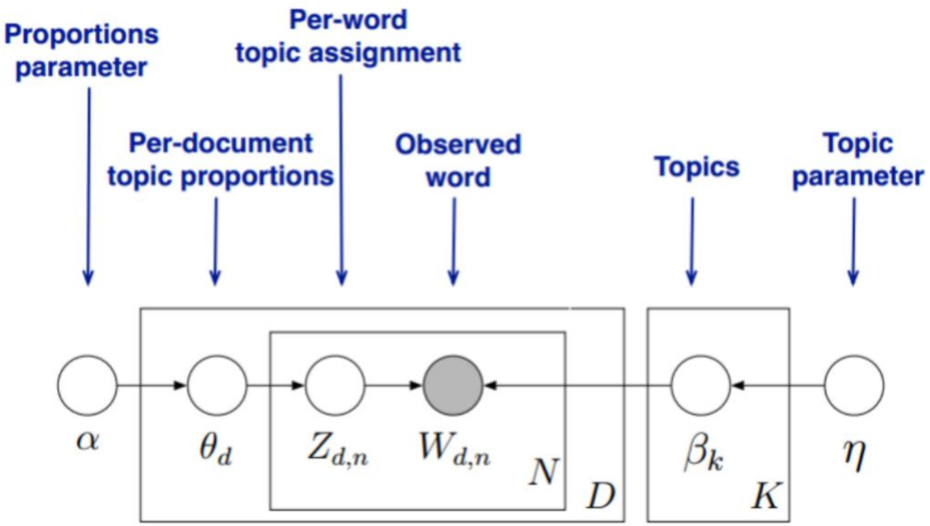
3. For each user-item pair (i, j), draw the response

$$r_{ij} \sim \mathcal{N}(u_i^T v_j, c_{ij}^{-1}),$$

where c_{ij} is the precision parameter for r_{ij}.

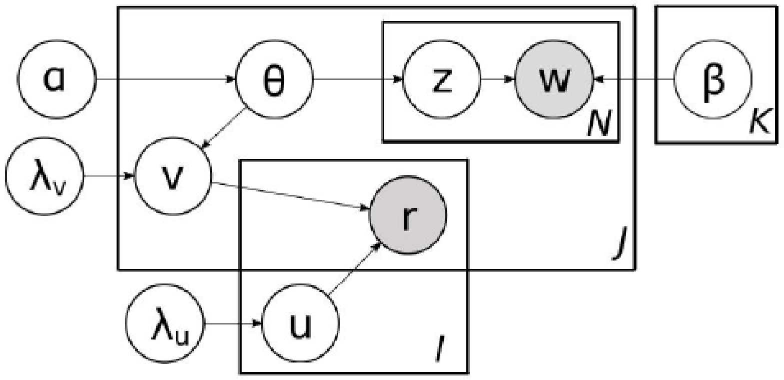
LDA

1. Draw topic proportions $\theta_j \sim \text{Dirichlet}(\alpha)$.
2. For each word n,
   (a) Draw topic assignment $z_{jn} \sim \text{Mult}(\theta_j)$.
   (b) Draw word $w_{jn} \sim \text{Mult}(\beta_{z_{jn}})$.



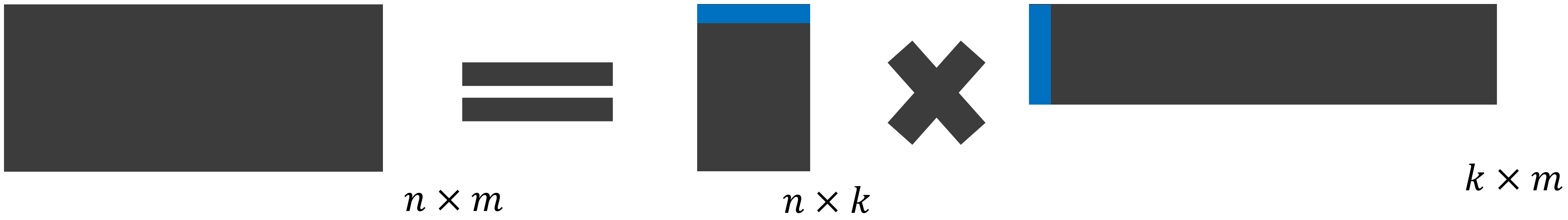
1. For each user i, draw user latent vector $u_i \sim \mathcal{N}(0, \lambda_u^{-1} I_K)$
2. For each item j;
   (a) Draw topic proportions $\theta_j \sim \text{Dirichlet}(\alpha)$
   (b) Draw item latent offset $\epsilon_j \sim \mathcal{N}(0, \lambda_v^{-1} I_K)$ and set the item latent vector as $v_j = \epsilon_j + \theta_j$
   (c) For each word w_{jn},
      i. Draw topic assignment $z_{jn} \sim \text{Mult}\ (\theta)$
      ii. Draw word $w_{jn} \sim \text{Mult}(\beta_{z_{jn}})$
3. For each user-item pair (i,j), draw the rating

$$r_{ij} \sim \mathcal{N}(u_i^T v_j, c_{ij}^{-1})$$

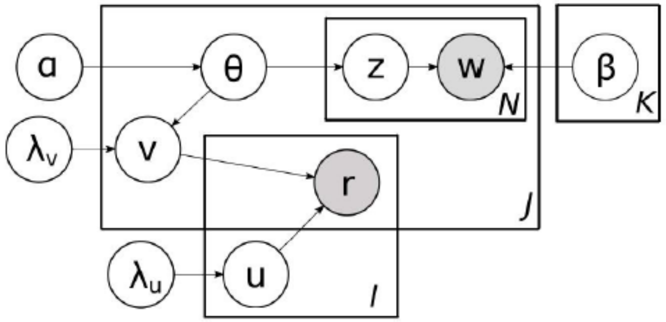


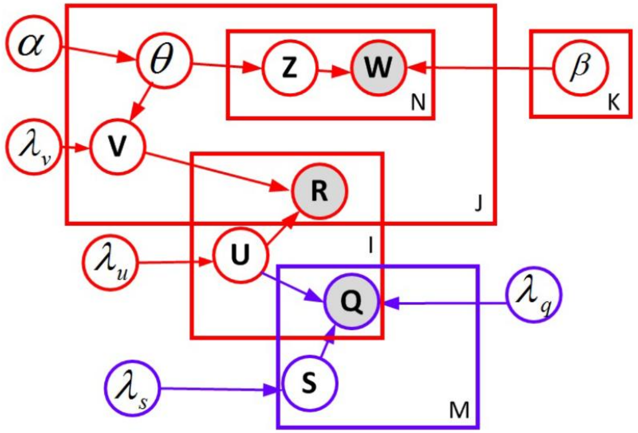
$$\mathbb{E}[r_{ij} | u_i, \theta_j, \epsilon_j] = u_i^T (\theta_j + \epsilon_j)$$

The key property in CTR lies in how the item latent vector v_j is generated, we assume the item latent vector v_j is close to topic proportions θ_j, but could diverge from it if it has to.



Collaborative Topic Regression with Social Matrix Factorization for Recommendation Systems ICML2012





$$P(Q|U,S,\sigma_Q^2)=\prod_{i=1}^{m}\prod_{k=1}^{m}\mathcal{N}(q_{ij}|g(U_i^TS_k),\sigma_Q^2)^{I_{ij}^Q}$$

$$P(U|\sigma_U^2)=\prod_{i=1}^{m}\mathcal{N}(U_i|0,\sigma_U^2 I) \qquad P(S|\sigma_S^2)=\prod_{k=1}^{m}\mathcal{N}(S_k|0,\sigma_S^2 I)$$

$$p(U,S|Q,\sigma_Q^2,\sigma_U^2,\sigma_S^2)\propto p(Q|U,S,\sigma_Q^2)p(U|\sigma_U^2)p(S|\sigma_S^2)$$

$$\begin{aligned}p(U,V,S|Q,R,\sigma_Q^2,\sigma_R^2,\sigma_U^2,\sigma_V^2,\sigma_S^2)\\ \propto p(R|U,V,\sigma_R^2)p(Q|U,S,\sigma_Q^2)\\ \times p(U|\sigma_U^2)p(V|\sigma_V^2)p(S|\sigma_S^2).\end{aligned}$$

Prediction

$$\mathcal{E}[r_{ij}|D]\approx\mathcal{E}[u_i|D]^T(\mathcal{E}[\theta_j|D]+\mathcal{E}[\epsilon_j|D])$$
$$r_{ij}^*\approx(u_i^*)^Tv_j^*$$

$$\mathcal{E}[r_{ij}|D]\approx\mathcal{E}[u_i|D]^T(\mathcal{E}[\theta_j|D])$$
$$r_{ij}^*\approx(u_i^*)^T\theta_j^*$$

$$P(V|\sigma_V^2)\sim\mathcal{N}(\theta_j,\lambda_V^{-1}I_k)$$