

Cost-Effective Active Learning for Hierarchical Multi-Label Classification

$$\min_{\alpha} \quad -tr\big((UH \square P_1)\alpha\big) + \lambda c\big(P_2 \square \alpha\big) \mathbf{1} + \|H - H_0\|^2$$

$$\min_{\alpha} \quad -tr\left((UH \begin{matrix} \square \\ \uparrow \end{matrix} P_1)\alpha\right) + \lambda c(P_2 \begin{matrix} \square \\ \downarrow \end{matrix} \alpha)\mathbf{1} + \|H - H_0\|^2$$

$$I = UH_0 = \begin{cases} U_{anc} + U, & f == 1 \\ U_{des} + U, & f == -1 \end{cases}$$

$$I_j = Uh_j = \begin{cases} U_{anc}^j + U_j, & f_j == 1 \\ U_{des}^j + U_j, & f_j == -1 \end{cases}$$

$$= \begin{cases} U_j + U_j, & f_j == 1 \\ \sum_{k \in leaf(j)} U_k + U_j, & f_j == -1 \end{cases}$$

$$h_j = \begin{bmatrix} 0 \\ \vdots \\ 2 \\ \vdots \\ 0 \end{bmatrix}$$

← j → H_1

$$h_j = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

← j → H_2

← k

$$\text{tr}((UH - P_1)\alpha)$$



$$\sum_{i=1}^n U_i (H_1 Y_i + H_2 \bar{Y}_i) - P_{1i} \alpha_i$$

$$Y_i = \begin{bmatrix} y_{i1} & & \\ & \dots & \\ & & y_{il} \end{bmatrix}$$

$$\bar{Y}_i = 1 - Y_i$$

$$\min_{\alpha} -\text{tr}((UH_0 - P_1)\alpha) + \lambda c(P_2 - \alpha) \mathbf{1}$$



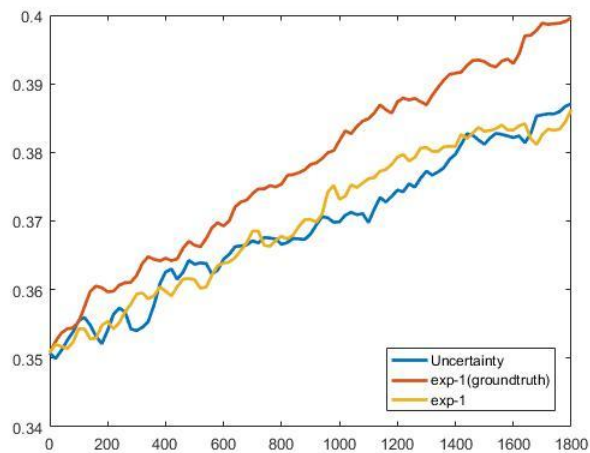
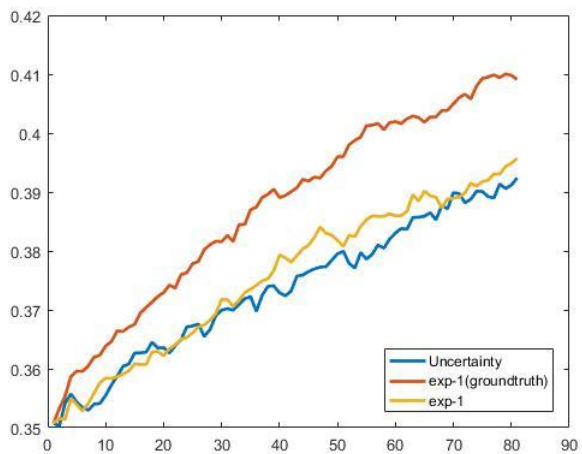
$$\min_{\alpha} -\sum_{i=1}^n (U_i (H_1 Y_i + H_2 \bar{Y}_i) - P_{1i}) \alpha_i + \lambda c(P_2 - \alpha) \mathbf{1}$$



$$\min_{\alpha} -\text{tr}((I - P_1)\alpha) + \lambda c(P_2 - \alpha) \mathbf{1}$$

$$\min_{\alpha} -tr((I \square P_1)\alpha)$$

$$I = \mathbb{I}[h = 1] * U_{anc} + \mathbb{I}[h = -1] * U_{des} + U$$



Yeast-go

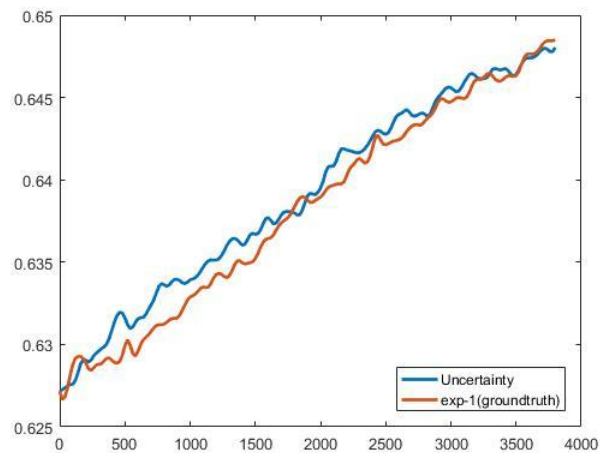
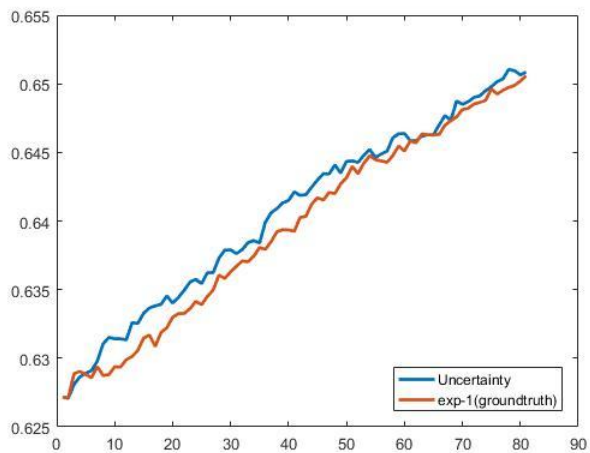
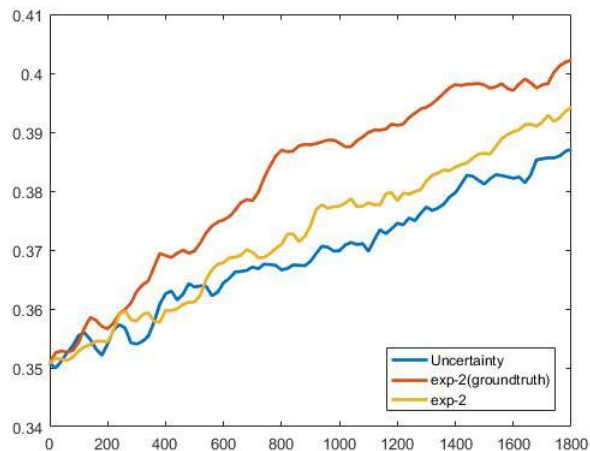
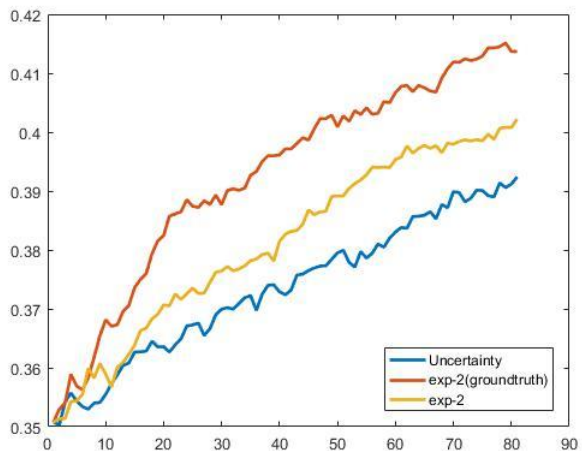


Image07D

$$\min_{\alpha} -tr((I \square P_1)\alpha) + \lambda c(P_2 \square \alpha) \mathbf{1}$$

$$I = \mathbb{I}[h = 1] * U_{anc} + \mathbb{I}[h = -1] * U_{des} + U$$



Yeast-go

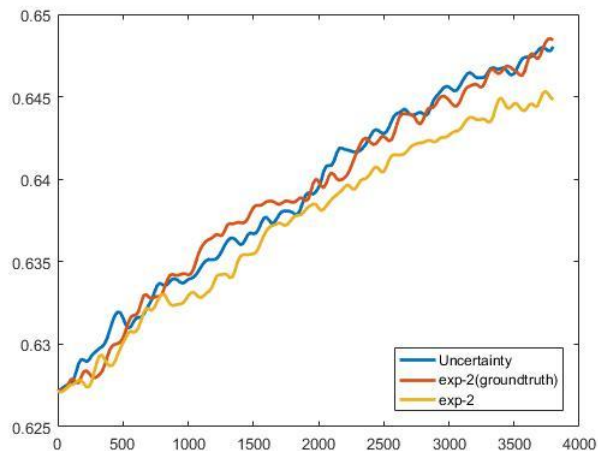
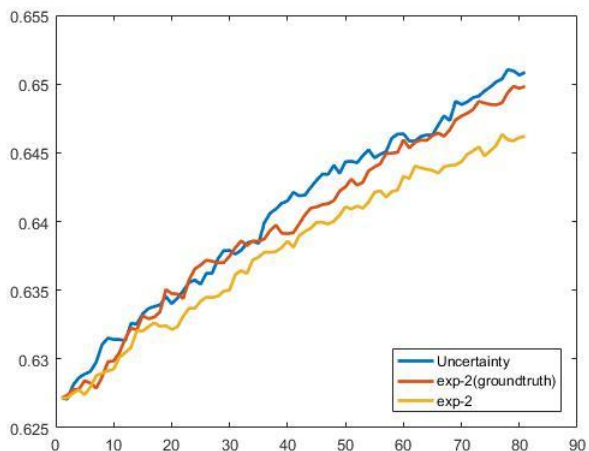


Image07D

$$\min_{\alpha} -tr((UH \square P_1)\alpha) + \lambda c(P_2 \square \alpha) \mathbf{1} + \|H - H_0\|^2$$



$$\min_{\alpha} -\sum_{i=1}^n (U_i(H_1 Y_i + H_2 \bar{Y}_i) \square P_{1i}) \square \alpha_i + \lambda c(P_2 \square \alpha) \mathbf{1} + \|H_1 - \bar{H}_1\|^2 + \|H_2 - \bar{H}_2\|^2$$